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GENERALIZED POWERED FLIGHT TRAJECTORY PROGRAM FOR IBM 704 COMPUTER

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ABSTRACT

The Powered Flight Trajectory Program, GNG06, is a digital computer program written in the SHARE language for the IBM 704, 32K machine. This generalized program computes the trajectory of a multi-stage rocket vehicle over an oblate spheroidal, rotating earth with atmosphere, in a launch-centered rectangular coordinate system. Path control of the vehicle is obtained by simple control from a selection of thirteen common path control methods. Included in the program are transformation of position and velocity components to a space-fixed rectangular coordinate system and also to local polar coordinate systems entered at observation stations. At option is the computation of the conic parameters of the solution to the two-body problem. The program contains differential correction routines for carrying out searches on one- or two dependent variables.

I. INTRODUCTION

The Powered Flight Trajectory Program is designed to provide a generalized computer program for trajectory studies in three dimensions. It is written in the SAP language for use with the IBM 704 computer and utilizes subroutines of the SHARE library. Its generalized nature allows a large variety of rocket vehicles to be described in single-stage or multi-stages under various modes of path control. Because of its generalized nature, the program may require a great amount of input data. However, in many cases, the mathematical description has been written so as to require minimum amounts of input data. The output format is arranged to give a complete description of the resulting trajectory in several convenient coordinate systems. The program may be operated from binary deck or from tape, once the tape is written. Furthermore, control has been included so that a powered flight trajectory may be terminated in this program and continued automatically in an N-Body Coast Trajectory Program, DBH06 (1).

A rectangular coordinate system is oriented with respect to an ellipsoidal, rotating earth at an initial time. The equations of motion are written in this coordinate system. The coordinate system remains inertial with respect to the rotation of the Earth. Transformations give position, velocity, and acceleration in Earth-fixed polar coordinates as well as Earth-centered, equatorial rectangular space-fixed coordinates. A standard atmosphere is included and is assumed to rotate uniformly with the Earth. Performance data describe the vehicle as an N-stage device, giving thrust, mass, drag, and lift force information for each stage. Path control is accomplished by imposing restrictions on the thrust vector in several available ways. Powered-flight stages and coast periods may be intermixed arbitrarily.

II. EQUATIONS OF MOTION

A. Coordinate Systems

An x_p, y_p, z_p rectangular coordinate system is established with origin at the geodedic latitude of φ_0' and longitude λ_0 at a height above the ellipsoidal earth, h_0. The y_p axis is perpendicular to the local horizontal plane (see Fig. 1). The x_p axis lies in this horizontal plane at an angle from true north of σ_L. The plane x_p, y_p is called the pitch plane. The z_p axis completes the right-handed system. A second coordinate system $\overset{''}{x}_p, \overset{''}{y}_p, \overset{''}{z}_p$ is given at the center of the Earth and is parallel to the x_p, y_p, z_p system. A third system, X, Y, Z is also given with its origin at the center of the Earth. The X axis lies in the equatorial plane and is directed toward the vernal equinox of date, while the Y axis is perpendicular to and east of X in the equatorial plane. The Z axis lies along the spin axis of the earth in the direction of north. This coordinate system is used to fix the trajectory in space in calendar time. Hence, the space relation of the trajectory with respect to other interplanetary bodies is easily determined, since ephemeris data giving the positions of these bodies as a function of calendar time are also given in this coordinate system. Although such ephemeris data are not included in this program, the position and velocity components in this space fixed coordinate system is of value for subsequent interplanetary trajectory studies.

Other coordinate systems used in the program will be described when their mathematical formulations are given. The above three systems are the fundamental systems used in describing the equations of motion, and hence are given at this time.

B. The International Ellipsoid

The oblate spheroidal earth is characterized by the semi-major and semi-minor axes and eccentricity of the elliptic section of the Earth. These quantities, a, b, ϵ, are given in Ref. 2 and have the following values:

$$a = 6378.388 \text{ km}$$

$$b = 6356.912 \text{ km}$$

$$\epsilon = 0.0819917861$$

Also, the earth angular rotation rate is given as ω, where

$$\omega = 0.7292116 \times 10^{-4} \text{ rad sec}$$

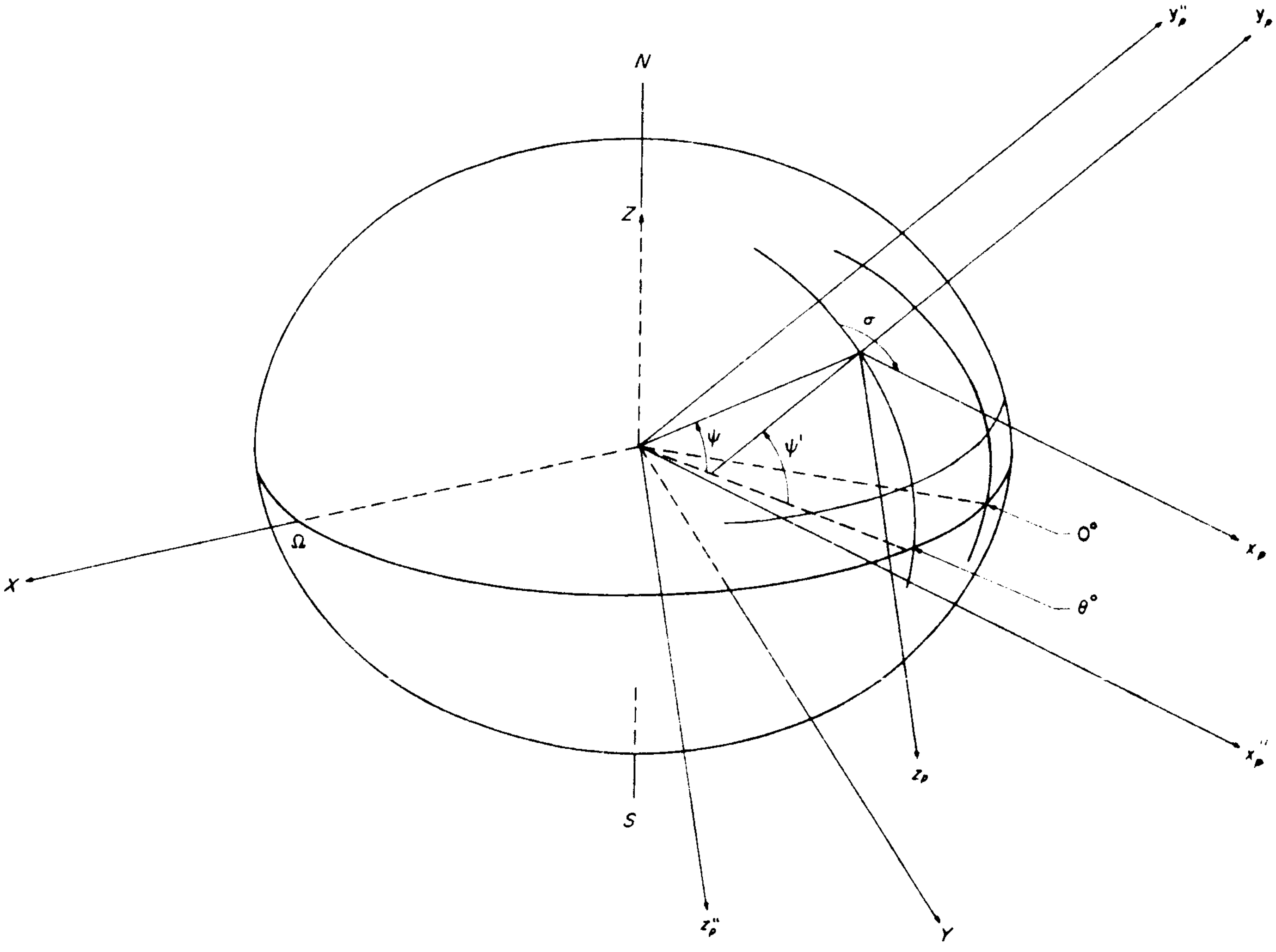


Fig. 1. Power flight program – GNG06 coordinate systems

C. Initial Conditions

To begin the computation, certain initial quantities must be computed. Several options are available in the form of these start conditions and therefore will be described separately. In any option, the following quantities are given initially:

t_0 = initial time with respect to launch

ψ_0' = geodedic latitude

λ_0 = longitude

σ_L = azimuth

h_0 = initial height

Also, the following computations are needed. Begin by finding the geocentric latitude of the origin of x_p, y_p, z_p coordinate system.

$$\left.\begin{aligned} x_c' &= \left(h_0 + \frac{a}{\sqrt{1 - (\epsilon \sin \psi_0')^2}}\right) \cos \psi_0' \\ y_c' &= \left(h_0 + \frac{(1 - \epsilon^2)\, a}{\sqrt{1 - (\epsilon \sin \psi_0')^2}}\right) \sin \psi_0' \\ \sin \psi_0 &= \frac{y_c'}{\sqrt{x_c'^2 + y_c'^2}} \end{aligned}\right\} \tag{1}$$

where ψ_0 is the geocentric latitude. The radius of the Earth at this geocentric latitude is

$$r_s = \frac{b}{\sqrt{1 - \epsilon^2 \cos^2 \psi_0}} \tag{2}$$

and the initial Earth-center-to-origin distance is

$$r_0 = h_0 + r_s \tag{3}$$

The difference between geodedic and geocentric latitude is given by

$$\beta_0 = \psi_0' - \psi_0 \tag{4}$$

Letting

$$\kappa = \frac{3\pi}{2} - \sigma_L$$

gives the coordinates of the origin in the x_p'', y_p'', z_p'' system at the center of the Earth:

$$\left.\begin{aligned} x_p''(0) &= r_0 \sin\kappa \sin\beta_0 \\ y_p''(0) &= r_0 \cos\beta_0 \\ z_p''(0) &= -r_0 \cos\kappa \sin\beta_0 \end{aligned}\right\} \tag{5}$$

The unit Earth spin-axis vector, $\vec{\Omega}'$, defined in the x_p, y_p, z_p system has components

$$\left.\begin{aligned} \Omega_1' &= -\cos\psi_0' \sin\kappa \\ \Omega_2' &= \sin\psi_0' \\ \Omega_3' &= \cos\psi_0' \cos\kappa \end{aligned}\right\} \tag{6}$$

where the spin-axis vector $\vec{\Omega}$ is given by

$$\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$$

where

$$\left.\begin{aligned} \Omega_1 &= \omega\Omega_1' \\ \Omega_2 &= \omega\Omega_2' \\ \Omega_3 &= \omega\Omega_3' \end{aligned}\right\} \tag{6a}$$

1. Option I: Start at Origin

The computation may begin at the origin of the coordinate system with a given Earth-fixed velocity v_0 and a pitch angle χ_0 measured from the x_p axis in the pitch plane. Then

$$
\left.
\begin{aligned}
x_p(t_0) = y_p(t_0) = z_p(t_0) &= 0 \\
\text{and} \qquad \qquad \\
\dot{x}_p(t_0) &= -\omega r_0 \cos \psi_0 \cos \kappa + v_0 \cos \chi_0 \\
\dot{y}_p(t_0) &= 1 + v_0 \sin \chi_0 \\
\dot{z}_p(t_0) &= -\omega r_0 \cos \psi_0 \sin \kappa
\end{aligned}
\right\} \tag{7}
$$

2. Option II: Start at $P\left[x_p(t_0), y_p(t_0), z_p(t_0)\right]$

In this option, the initial point in the coordinate system is given by the values

$$x_p(t_0), y_p(t_0), z_p(t_0)$$

and the velocity components by the values

$$\dot{x}_p(t_0), \dot{y}_p(t_0), \dot{z}_p(t_0)$$

These quantities are substituted explicitly for Eq. (7), and the computation begins at this point.

3. Option III: Start at $P(r, v_e, \psi, \lambda, \odot, \sigma)$

An initial point is given by its Earth-fixed polar coordinates and Earth-fixed velocity and pitch angle. It is now necessary to transform these quantities into position and velocity components (such as the explicit position and velocity components in Option II). Let the following definitions hold:

r = Earth center to vehicle distance

v_e = Earth-fixed velocity

ψ = geocentric latitude

λ = longitude

Θ = Earth-fixed path angle

σ = Earth-fixed azimuth angle

From the calendar date, the Julien Date (JD) may be found. Then the approximation

$$\mathrm{GHA}(t_m) = 258.572200988 + 0.985647543\, T \tag{8}$$

where $T = \mathrm{JD} - 2436000.0$ and $\mathrm{GHA}(t_m)$ is the Greenwich Hour Angle at midnight of the day JD. The right ascension of the origin (or launcher) at the time of launch t_L, measured with respect to midnight, is[1]

$$\Theta_L = \mathrm{GHA}(t_m) + \omega t_L + \lambda_0 \tag{9}$$

The origin of the x_p, y_p, z_p system is now defined in the Earth-centered equatorial coordinate system (see Sec. II-A). It is given by

$$\left.\begin{aligned} X_L &= r_0 \cos \Theta_L \cos \psi_0 \\ Y_L &= r_0 \sin \Theta_L \cos \psi_0 \\ Z_L &= r_0 \sin \psi_0 \\ &\text{The velocity components, since both systems are inertial with respect to rotation, are} \\ \dot{X}_L &= \dot{Y}_L = \dot{Z}_L = 0 \end{aligned}\right\} \tag{10}$$

The Greenwich Hour Angle at the initial time of computation, t_0, is found from the GHA (t_m) by

$$\mathrm{GHA}(t_0) = \mathrm{GHA}(t_m) + \omega(t_L + t_0) \tag{11}$$

and the right ascension of the vehicle at this time is

$$\Theta = \mathrm{GHA}(t_0) + \lambda \tag{12}$$

[1]Note that t_L is the launch time in sec referenced to midnight of the date, while t_0 (and also, as will be seen later, t) is the time in sec after launch.

In a manner similar to that given by Eq. (10), the space-fixed coordinates of the vehicle at the initial time are given by

$$
\begin{aligned}
X &= r \cos \Theta \cos \psi \\
Y &= r \sin \Theta \cos \psi \\
Z &= r \sin \psi
\end{aligned}
\tag{13}
$$

A new coordinate system is used as an auxiliary to find the space-fixed velocity components. This system is Earth-centered with the x axis in the equatorial plane at 0 deg longitude, y axis also in the equatorial plane perpendicular and east of the x axis, and the z axis directed north on the spin axis. This coordinate system rotates with the Earth. The position of the vehicle in this system is

$$
\left.
\begin{aligned}
x &= r \cos \psi \cos \lambda \\
y &= r \cos \psi \sin \lambda \\
z &= r \sin \psi
\end{aligned}
\right\}
\tag{14}
$$

The velocity components in this system are described by the following set of transformations:

$$
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi & 0. & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\sigma & 0 \\ 0 & 0 & \cos\sigma \end{bmatrix} \begin{bmatrix} v_e & \sin & \circleddash \\ v_e & \cos & \circleddash \\ v_e & \cos & \circleddash \end{bmatrix} \tag{15}
$$

Finally, the velocity components of the vehicle in the space-fixed system are given by the following transformation:

$$
\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \cos \mathrm{GHA}(t_0) & -\sin \mathrm{GHA}(t_0) & 0 \\ \sin \mathrm{GHA}(t_0) & \cos \mathrm{GHA}(t_0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} - \omega y \\ \dot{y} + \omega x \\ \dot{z} \end{bmatrix} \tag{16}
$$

Another transformation is required to find the x_p, y_p, z_p coordinates from the space fixed coordinates. This is found from the product of three rotations as

$$\begin{vmatrix} U_I' & U_J' & U_K' \\ V_I' & V_J' & V_K' \\ W_I' & W_J' & W_K' \end{vmatrix} = \begin{vmatrix} \sin\sigma_L & 0 & -\cos\sigma_L \\ 0 & 1 & 0 \\ \cos\sigma_L & 0 & \sin\sigma_L \end{vmatrix} \cdot \begin{vmatrix} -1 & 0 & 0 \\ 0 & \cos(\beta_0+\psi_0) & -\sin(\beta_0+\psi_0) \\ 0 & \sin(\beta_0+\psi_0) & \cos(\beta_0+\psi_0) \end{vmatrix} \cdot \begin{vmatrix} \sin\Theta & -\cos\Theta & 0 \\ \cos\Theta & \sin\Theta & 0 \\ 0 & 0 & -1 \end{vmatrix} \tag{17}$$

The initial position and velocity components are now given by

$$\left.\begin{aligned} \begin{vmatrix} x_p \\ y_p \\ z_p \end{vmatrix} &= \begin{vmatrix} U_I' & U_J' & U_R' \\ V_I' & V_J' & V_R' \\ W_I' & W_J' & W_R' \end{vmatrix} \begin{vmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{vmatrix} \\ \begin{vmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{vmatrix} &= \begin{vmatrix} U_I' & U_J' & U_R' \\ V_I' & V_J' & V_R' \\ W_I' & W_J' & W_R' \end{vmatrix} \begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{vmatrix} \end{aligned}\right\} \tag{18}$$

Option III is then complete, with $x_p(t_0)$, $y_p(t_0)$, $z_p(t_0)$ and corresponding velocities having been found from the given Earth-fixed polar coordinates.

D. Equations of Motions

The equations of motion in the fundamental coordinate system are as follows:

$$\left.\begin{aligned} \ddot{x}_p &= g_1 + \ddot{x}_m \\ \ddot{y}_p &= g_2 + \ddot{y}_m \\ \ddot{z}_p &= g_3 + \ddot{z}_m \\ \text{where} \quad \ddot{x}_m &= (F/M) f_1 - (A_f/M) c_1 + (N/M) n_1 \\ \ddot{y}_m &= (F/M) f_2 - (A_f/M) c_2 + (N/M) n_2 \\ \ddot{z}_m &= (F/M) f_3 - (A_f/M) c_3 + (N/M) n_3 \end{aligned}\right\} \tag{19}$$

The quantities in Eq. (19) will now be defined. From the results of Eq. (5), the Earth-centered coordinates are found by

$$\begin{aligned} x''_p &= x_p + x''_p(0) \\ y''_p &= y_p + y''_p(0) \\ z''_p &= z_p + z''_p(0) \end{aligned} \tag{20}$$

1. **Earth-Related Quantities**

The Earth-center-to-vehicle distance is now found by

$$r = \sqrt{x''^2_p + y''^2_p + z''^2_p} \tag{21}$$

with the unit vector along r given by $\vec{r}'$ where

$$\left. r'_1 = \frac{x''_p}{r}, \qquad r'_2 = \frac{y''_p}{r}, \qquad r'_3 = \frac{z''_p}{r} \right\} \tag{22}$$

Then $\sin \psi = \vec{r}' \cdot \vec{\Omega}'$ where ψ is the geocentric latitude.

From the potential equation [see Ref. 7, Eq. (58), (59)]

$$A' = -32.146619 \left(\frac{a}{r}\right)^2 - 0.052661 \left(\frac{a}{r}\right)^4 - 0.000148 \left(\frac{a}{r}\right)^6$$

$$+ 0.263301 \left(\frac{a}{r}\right)^4 \sin^2 \psi + \left(\frac{a}{r}\right)^6 (-0.002057 \sin^4 \psi + 0.003077 \sin^2 \psi)$$

$$B' = -0.105319 \left(\frac{a}{r}\right)^4 \sin \psi + \left(\frac{a}{r}\right)^6 (0.001355 \sin^2 \psi - 0.000581) \sin \psi \tag{23}$$

The components of acceleration due to gravity are

$$\left.\begin{aligned} g_1 &= A'r_1' + B'\Omega_1' \\ g_2 &= A'r_2' + B'\Omega_2' \\ g_3 &= A'r_3' + B'\Omega_3' \end{aligned}\right\} \tag{24}$$

Since atmospheric drag and normal force are functions of Earth-fixed velocity, it is necessary to find the components of Earth-fixed velocity in the inertial coordinate system. These are

$$\begin{aligned} v_{1p} &= \dot{x}_p - (\Omega_2 \ddot{z}_p - \Omega_3 \ddot{y}_p) \\ v_{2p} &= \dot{y}_p - (\Omega_3 \ddot{x}_p - \Omega_1 \ddot{z}_p) \\ v_{3p} &= \dot{z}_p - (\Omega_1 \ddot{y}_p - \Omega_2 \ddot{x}_p) \end{aligned}$$

and the total Earth-fixed velocity is

$$v_e = \sqrt{v_{1p}^2 + v_{2p}^2 + v_{3p}^2} \tag{25}$$

with the unit Earth-fixed velocity vector, $\vec{v} = (v_1', v_2', v_3')$, where

$$v_1' = \frac{v_{1p}}{v_e}, \qquad v_2' = \frac{v_{2p}}{v_e}, \qquad v_3' = \frac{v_{3p}}{v_e}$$

Using the geocentric latitude from Eq. (22) in Eq. (2), the radius of the Earth for this latitude is found for determining the height

$$h = r - r_s \tag{26}$$

2. Atmosphere

Pressure ratio, $p(h)/p(0)$, and accoustic velocity, $a(h)$, are now needed to find quantities that are functions of atmosphere. The Power Flight Trajectory Program includes the above quantities taken from the ARDC Standard. Atmosphere Table, 1957, where $p(h)/p(0)$ and $a(h)$ have been fit to polynomials. (See Appendix III.) They are available for -2000 ft $\leq h \leq$ 300000 ft. For $300000 < h < 10^6$, the following extrapolation function is used:

$$\left.\begin{aligned}
&\log_{10}\frac{p(h)}{p(0)} = \frac{2116261.17}{h} + 0.18825055 - 13 \\
&a(h) = 1100.0 \text{ ft/sec} \\
&\text{For } h > 10^6, \\
&\frac{p(h)}{p(0)} = 0 \\
&a(h) = 1100.0
\end{aligned}\right\} \tag{27}$$

3. Performance Quantities

At this point, certain vehicle performance data are required. These are defined as follows:

F_0 = vacuum thrust (lb)

f_e = exhaust area (ft^2)

W_g = gross weight (lb)

W_e = empty weight (lb)

W_f = fuel weight (lb)

$\dot{W}_p$ = mass flow (lb/sec)[2]

These data are provided for each stage of the vehicle where W_g is the total weight of the entire vehicle at the beginning of a particular stage. The total thrust is found by

$$F = F_0 - f_e\, p(h) \tag{28}$$

and the mass by

$$m(t) = \frac{W(t)}{g_0} = \left(W_g - \int_{t_i}^{t} \dot{W}_p\, dt\right)\frac{1}{g_0} \tag{29}$$

[2] Mass converted by $g = 32.172$

where $g_0 = 32.172$, t_i = initial time for the stage. An option allows both F_0 and $\dot{W}_p$ to be represented by 6th-degree polynomials by proper program control. Coasting is attained by setting $F = 0$ and

$$m(t_b) = \left(W_g - \int_{t_i}^{t_b} \dot{W}_p \, dt\right) \frac{1}{g_0} \tag{29a}$$

This condition is maintained until new data are required for the following stage. In the following stage W_g may be provided explicitly, or W_g may be computed from weight at this time. That is, for the next stage, W_g may be

$$W_g = W(t_b) - W_e \tag{29b}$$

where t_b is time of burnout of the previous stage and W_e is the weight that is discarded. A relation

$$p(t) = w(t) - W_e \tag{29c}$$

may be computed for all t. At the end of a complete trajectory, $p(t)$ is the payload weight.

The shutoff of any stage may be controlled in a number of ways (see Sec. III-D), one of which is based on the computation of weight. Let

$$\overline{W}_f = \int_{t_i}^{t} \dot{W}_p \, dt \tag{30}$$

Given the quantity W_f, shutoff occurs when

$$\overline{W}_f = W_f$$

4. Drag

Atmospheric drag forces may be computed using the expression for drag coefficient, C_{d_0}, and the effective diameter, d, of the vehicle. The program allows two options in presenting C_{d_0}. A constant for each stage, C_{d_i} may be introduced which is defined as follows:

$$C_{d_0} = \frac{4C_{d_i}}{\pi} \tag{31}$$

If C_{d_i} is set equal to zero, then the drag coefficient has the following definition:

$$C_{d_0} = \left\{ \begin{array}{ll} C_{0_1} + C_{1_1}M + C_{2_1}M^2 + C_{3_1}M^3 & 0 \leq M < m_1 \\ C_{0_2} + C_{1_2}M + C_{2_2}M^2 + C_{3_2}M^3 & m_1 \leq M < m_2 \\ C_{0_3} + C_{1_3}M + C_{2_3}M^2 + C_{3_3}M^3 & m_2 \leq M < m_3 \\ C_{0_4} + C_{1_4}M + C_{2_4}M^2 + C_{3_4}M^3 & m_3 \leq M < m_4 \\ C_{0_5} + C_{1_5}M + C_{2_5}M^2 + C_{3_5}M^3 & m_4 \leq M \end{array} \right\} \tag{32}$$

where Mach number, M, is given by

$$M = \frac{v_e}{a(h)} \tag{33}$$

The dynamic pressure is found from

$$q = \frac{\gamma_0}{2} P(h) M^2 \tag{34}$$

and the axial drag force is now found by

$$A_f = \frac{\pi}{4} C_{d_0} q d^2 \tag{35}$$

5. Path Direction

At this point, some quantities may be determined that are not explicitly required to solve the differential equations. They are, however, of interest in trajectory computation. First, the angle may be found that the Earth-fixed velocity vector makes with the local horizontal plane.

$$\sin \Theta = \vec{r} \cdot \vec{v} \tag{36}$$

where $\vec{r}$ and $\vec{v}$ were given by Eq. (22) and (25), respectively. The comparable angle for the inertial velocity vector is

$$\sin \gamma = \vec{r} \cdot \vec{v}_i$$

with

$$\vec{v}_i = (v'_{i_1}, v'_{i_2}, v'_{i_3})$$

and

$$v'_{i_1} = \frac{\dot{x}_p}{v_i}, \quad v'_{i_2} = \frac{\dot{y}_p}{v_i}, \quad v'_{i_3} = \frac{\dot{z}_p}{v_i} \tag{37}$$

where

$$v_i = \sqrt{\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2}$$

Consider the projection of the Earth-fixed velocity vector in the local horizontal plane. This projection makes an angle, σ, with north, and hence is the Earth-fixed aximuth angle. For proper quadrant definition, this angle is given by the expressions

$$\sin \sigma = \frac{\vec{\Omega} \cdot (\vec{r} \times \vec{v})}{\cos \psi \cos \odot}$$

$$\cos \sigma = \left[\left\{ \vec{r} \times \left(\frac{\vec{\Omega} \times \vec{r}}{\cos \psi} \right) \right\} \times \vec{r} \right] \cdot \left[\frac{\vec{v} \times \vec{r}}{\cos \odot} \right] \tag{38}$$

where

$$\cos \psi \neq 0, \quad \cos \odot \neq 0$$

The comparable angle for the inertial velocity vector is

$$\cos \sigma_i = \frac{v_e \cos \odot \cos \sigma}{v \cos \gamma} \tag{39}$$

6. Path Control

Two vectors are now introduced and defined. The $\vec{c}$ vector is defined as a unit vector aligned along the longitudinal axis of the vehicle, and has components (c_1, c_2, c_3) in the coordinate system. Similarly, the thrust vector, $\vec{f} = (f_1, f_2, f_3)$, is the unit vector that points in the direction of thrust. For simplicity, the assumption is made that $\vec{c} = \vec{f}$; that is, the thrust vector is constrained to point along the longitudinal axis of the vehicle. It remains, then, to apply adequate constraints upon the $\vec{c}$ vector in order that path control may be achieved. To do this, we define two angles, the 'pitch' angle, χ, and the 'yaw' angle, τ.

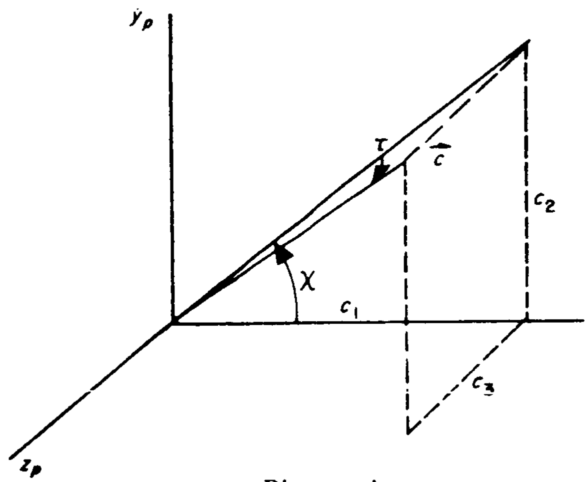


Diagram A

From Diagram A, it is evident that the following relations are true:

$$c_1 = \cos\chi \cos\tau$$

$$c_2 = \sin\chi \cos\tau$$

$$c_3 = \sin\tau \qquad (40)$$

Control then is imposed by either explicitly defining $\vec{c}$ or by defining the angles χ and τ. The set of options on path control that is available in the Power Flight Trajectory Program is given below. Angles of attack, as given in the following list, are defined by Diagram B.

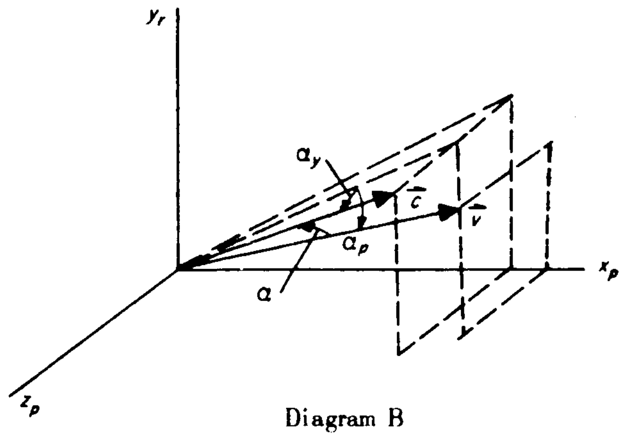


Diagram B

1. Control by given angle of attack in pitch, α_p:

$$\left.\begin{aligned} \sin\chi &= \frac{v_1' \sin\alpha_p + v_2' \sqrt{\cos^2\alpha_p - v_3'^2}}{v_1'^2 + v_2'^2} \\ \cos\chi &= \frac{-v_2' \sin\alpha_p + v_1' \sqrt{\cos^2\alpha_p - v_3'^2}}{v_1'^2 + v_2'^2} \end{aligned}\right\} \tag{41}$$

2. Control by given angle of attack in yaw, α_y:

$$\left.\begin{aligned} \cos\gamma' &= \frac{v_3'}{\cos\alpha_p} \\ \tau &= \alpha_y + \gamma \end{aligned}\right\} \tag{42}$$

3. Zero lift:

$$\vec{c} = \vec{v} \tag{43}$$

4. Gravity turn:

$$\vec{c} = \vec{v}_i \tag{44}$$

5. Vertical flight:

$$\chi = \frac{\pi}{2}, \quad \tau = 0, \quad \alpha = 0 \tag{45}$$

6. Constant pitch angle:

$$\chi = \bar{\chi} \text{ where } \bar{\chi} = \text{given constant} \tag{46}$$

7. Pitch angle as function of time:

$$\chi = \bar{\chi}(t) \text{ where } \bar{\chi}(t) = \text{6th degree polynomial} \tag{47}$$

8. Modify pitch angle and hold constant:

Let $\chi(t_i) = \chi$ at some time t_i. Then set

$$\chi \text{ (constant)} = \chi(t_i) + \Delta\chi \tag{48}$$

given $\Delta\chi$.

9. Zero yaw angle:

$$\tau = 0 \tag{49}$$

10. No yaw restriction:

$$\sin \tau = v_3' \tag{50}$$

11. Yaw as function of time:

$$\tau = \bar{\tau}(t) \text{ where } \bar{\tau}(t) = \text{6th-degree polynomial} \tag{51}$$

12. Modify yaw angle and hold constant:

Let $\tau(t_i) = \tau$ at some time t_i. Then set

$$\tau \text{(constant)} = \tau(t_i) + \Delta\tau \tag{52}$$

given $\Delta\tau$.

13. Reference pitch angle to horizon: Given the angle, μ, measured from the local horizon.

$$\left.\begin{aligned} \left(\frac{\vec{r} \times \vec{v}_i}{\cos \Gamma}\right) \cdot \vec{c} &= 0 \\ \vec{r} \cdot \vec{c} &= \sin \mu \\ \vec{v}_i \cdot \vec{c} &= \cos (\gamma - \mu) \end{aligned}\right\} \tag{53}$$

Solve Eq. (53) simultaneously for $\vec{c}$.

This completes the set of controls given by the Powered Flight Trajectory Program. In all the above, if the angle of attack is not zero, then the total angle of attack is

$$\cos \alpha = \vec{c} \cdot \vec{v} \tag{54}$$

Also, if $\alpha \neq 0$, normal or lift force exists. This force acts in a direction perpendicular to the $\vec{c}$ vector and in the plane of α. If the normal force vector is $\vec{n} = (n_1, n_2, n_3)$, then

$$n_i = \frac{c_i \cos \alpha - v_i}{\sin \alpha} \qquad i = 1, 2, 3 \tag{55}$$

The total normal force is defined as being linear with α, and is given by

$$N = \frac{\pi}{4} C_z' q d^2 \alpha \tag{56}$$

where C_z', the lift coefficient must be given in a manner similar to the polynomials in Mach number of the defined C_{d_0} in Eq. (32). This completes the control equations.

7. Additional Quantities

At this point, enough information is available to form the right-hand side of the basic differential Eq. (19). There are, however, additional quantities that may be computed that are of interest. For example, the angle subtended at the center of the Earth from the origin of the coordinate system and the position at any later time, t, is given by

$$\cos \phi = \vec{r}_0 \cdot \vec{r} \tag{57}$$

The range over an average sphere is

$$R = \frac{r_0 + r}{2} \phi \tag{58}$$

The longitude may be found by first computing the change in longitude from t_0 to any time t.

$$\cos (\Delta\lambda) = \frac{(\vec{r}_0 \times \vec{\Omega}) \cdot (\vec{r} \times \vec{\Omega})}{\cos \psi_0 \quad \cos \psi} \tag{59}$$

Longitude is given by

$$\lambda = (\lambda_0 + \Delta\lambda - \omega t) \bmod 2\pi \tag{60}$$

The space-fixed, vernal equinox coordinate system described earlier is of interest for interplanetary trajectory studies. Hence a transformation from the launch centered inertial system to this space-fixed system is given. Let the right ascension of the origin of the launch centered system be

$$\Theta_L = \mathrm{GHA}(t_m) + \omega t_L + \lambda_0 \tag{61}$$

and let

$$\left.\begin{aligned}
&[H] = \begin{bmatrix} -\sin\Theta_L & -\cos\Theta_L & 0 \\ \cos\Theta_L & -\sin\Theta_L & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [G] = \begin{bmatrix} \Omega_7' & \Omega_8' & \Omega_9' \\ \Omega_4' & \Omega_5' & \Omega_6' \\ \Omega_1' & \Omega_2' & \Omega_3' \end{bmatrix} \\
&\text{where} \\
&\Omega_4' = -\sin\psi_0' \sin\kappa \qquad \Omega_7' = -\cos\kappa \\
&\Omega_5' = -\cos\psi_0' \qquad \Omega_8' = 0 \\
&\Omega_6' = \sin\psi_0' \cos\kappa \qquad \Omega_9' = -\sin\kappa
\end{aligned}\right\} \tag{62}$$

and Ω_i', $i = 1, 2, 3$ are given by Eq. (6).

Let $|C| = |H| \cdot |G|$. Then the space-fixed position, velocity, and acceleration components are as follows:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = |C| \cdot \begin{vmatrix} x''_p \\ y''_p \\ z''_p \end{vmatrix} \qquad \begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{vmatrix} = |C| \cdot \begin{vmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{vmatrix} \qquad \begin{vmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{vmatrix} = |C| \cdot \begin{vmatrix} \ddot{x}_p \\ \ddot{y}_p \\ \ddot{z}_p \end{vmatrix} \tag{63}$$

8. Velocity Shutoff Equation

In discussion of the mass computation, an option for shutoff on the basis of weight was discussed. Another basis for shutoff is given now. It is desired to shut off a stage when the measurable velocity attains a given standard value. The total measurable acceleration is given by

$$a_m = \sqrt{\ddot{x}_m^2 + \ddot{y}_m^2 + \ddot{z}_m^2} \tag{64}$$

where $\ddot{x}_m$, $\ddot{y}_m$, $\ddot{z}_m$ are given by Eq. (19).

The true measurable acceleration is that component of a_m that lies in the direction of the $\vec{c}$ vector. That is,

$$a_x = \vec{a}_m \cdot \vec{c} \tag{65}$$

Then, the true measurable velocity is defined by

$$V_x = \kappa_1 \int_{t_0}^{t} a_x \, dt + V_x(t_0) + \kappa_2 + \kappa_3(a' - t) + \kappa_4 (b' - t)^2 \tag{66}$$

where κ_i are gain factors, $V_x(t_0)$ the initial true measurable velocity, and $(a' - t)$, $(b' - t)$ are drift terms, all of which are given. Let

$$V_s = V_{x_s} - V_x \tag{67}$$

where V_{x_s} is the desired value of velocity. Then, when $V_s = 0$, shutoff occurs.

9. Observation Stations

It is of interest to view the trajectory from one or more points referenced to the rotating uniform ellipsoid. Letting these points be defined by R_i, ϕ_i, Θ_i, the distance from the center of the Earth, the geocentric latitude, and longitude, respectively, gives the following quantities with respect to this point:

$$R_s,\ \dot{R}_s,\ \ddot{R}_s = \text{slant range, rate, acceleration}$$

$$\alpha,\ \dot{\alpha} = \text{hour angle, rate}$$

$$\delta,\ \dot{\delta} = \text{declination, rate}$$

$$e,\ \dot{e} = \text{elevation, rate}$$

$$\sigma,\ \dot{\sigma} = \text{azimuth, rate}$$

$$L = \text{look angle}$$

$$p = \text{polarization angle}$$

Begin by defining an Earth-centered, rectangular coordinate system that has an x_e axis in the equatorial plane in the direction of the Greenwich meridian. The y_e axis is also in the equatorial plane and is 90 deg east of the x_e axis, while the z_e axis points north along the spin axis of the Earth. This coordinate system is Earth-fixed and is related to the space fixed system by the rotation T such that

$$\begin{vmatrix} X_e \\ Y_e \\ Z_e \end{vmatrix} = |T| \cdot \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}, \quad \begin{vmatrix} \dot{X}_e \\ \dot{Y}_e \\ \dot{Z}_e \end{vmatrix} = |T| \cdot \begin{vmatrix} \omega Y + \dot{X} \\ -\omega X + \dot{Y} \\ \dot{Z} \end{vmatrix}, \quad \begin{vmatrix} \ddot{X}_e \\ \ddot{Y}_e \\ \ddot{Z}_e \end{vmatrix} = |T| \cdot \begin{vmatrix} (\omega \dot{Y} + \ddot{X}) + (\omega^2 Y + \omega \dot{X}) \\ (-\omega \dot{X} + \ddot{Y}) + (-\omega^2 X + \omega \dot{Y}) \\ \ddot{Z} \end{vmatrix} \tag{68}$$

The rotation $|T|$ is simply

$$|T| = \begin{vmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{69}$$

where Θ is given by Eq. (12). The $\vec{c}$ vector is needed in the space fixed system, and is found by

$$\vec{C}_i = |C| \cdot \vec{c}$$

where $|C|$ was given by Eq. (63). Using the quantities that define the observation point, the position coordinates of this point in the above Earth-fixed system are given by

$$\left.\begin{aligned} X_i &= R_i \cos \phi_i \cos \bigodot_i \\ Y_i &= R_i \cos \phi_i \sin \bigodot_i \\ Z_i &= R_i \sin \phi_i \end{aligned}\right\} \tag{70}$$

Now the slant range, slant range rate, and slant range rate change may be found by the following equations:

$$R_s = [(X_e - X_i)^2 + (Y_e - Y_i)^2 + (Z_e - Z_i)^2]^{1/2} \tag{71}$$

$$\dot{R}_s = \frac{(X_e - X_i)\dot{X}_e + (Y_e - Y_i)\dot{Y}_e + (Z_e - Z_i)\dot{Z}_e}{R_s} \tag{72}$$

$$\ddot{R}_s = \frac{(X_e - X_i)\ddot{X}_e + (Y_e - Y_i)\ddot{Y}_e + (Z_e - Z_i)\ddot{Z}_e + \dot{X}_e^2 + \dot{Y}_e^2 + \dot{Z}_e^2 - \dot{R}_s^2}{R_s} \tag{73}$$

The hour angle, declination, elevation, azimuth, and their rates are found by the following set of equations:

$$\alpha = (\bigodot_i - \bar{\bigodot}) \bmod 2\pi$$

where

$$\bar{\bigodot} = \bar{\bigodot}' \bmod 2\pi$$

and

$$\cos \bar{\bigodot}' = \frac{X_e - X_i}{\sqrt{(X_e - X_i)^2 + (Y_e - Y_i)^2}} \tag{74}$$

$$\dot{\alpha} = \frac{-(X_e - X_i)\dot{Y}_e + (Y_e - Y_i)\dot{X}_e}{(X_e - X_i)^2 + (Y_e - Y_i)^2} \tag{75}$$

$$\sin \delta = \frac{Z_e - Z_i}{R_s} \qquad -\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2} \tag{76}$$

$$\dot{\delta} = \frac{\dot{Z}_e - \dot{R}_s \sin \delta}{R_s \cos \delta} \tag{77}$$

$$\sin e = \frac{(X_e - X_i) X_i + (Y_e - Y_i) Y_i + (Z_e - Z_i) Z_i}{R_i R_s} \tag{78}$$

$$\cos \sigma = \frac{-(X_e - X_i) \sin \phi_i \cos \odot_i - (Y_e - Y_i) \sin \phi_i \sin \odot_i + (Z_e - Z_i) \cos \phi_i}{R_s \cos e} \tag{79}$$

$$\dot{\sigma} = \frac{\dot{X}_e \cos \phi_i \cos \odot_i + \dot{Y}_e \sin \phi_i \sin \odot_i - \dot{Z}_e \cos \phi_i + (\dot{R}_s \cos e - \dot{e} R_s \sin e) \cos \sigma}{R_s \sin \sigma \cos e} \tag{80}$$

The look angle, or the angle that the $\vec{C}_i$ vector makes with the slant range vector, is given by

$$\cos L = \frac{(X - \overline{X}_i) C_{1_i} + (Y - \overline{Y}_i) C_{2_i} + (Z - \overline{Z}_i) C_{3_i}}{R_s} \tag{81}$$

where

$$\begin{vmatrix} \overline{X}_i \\ \overline{Y}_i \\ \overline{Z}_i \end{vmatrix} = |T|^{-1} \begin{vmatrix} X_i \\ Y_i \\ Z_i \end{vmatrix}. \tag{82}$$

The polarization angle, p, is defined in vector notation as follows:

$$\cos p = \frac{(\vec{c} \times \vec{r_i}) \cdot (\vec{r} \times \vec{r}_i)}{\cos(\vec{c},\vec{r}) \cdot \cos(\vec{r},\vec{r}_i)} \tag{83}$$

where $\vec{r_i}$ is the unit observation point vector.

Finally, given the station transmitter parameters A_i, B_i, C_i, D_i, and the velocity of light, c, the frequency received at the station is a function of the transmitted frequency, f_{c_0}, is

$$f_i = \frac{f_{c_0} + B_i + C_i - \left(\frac{f_{c_0}}{c}\right) \dot{R}_s}{A_i} \tag{84}$$

10. Conic Parameters

In addition to the above calculation, the program includes a set of equations representing the solution to the two-body problem involving the Earth and the vehicle. At any time, on control at input, the inertial velocity v_i, Earth-center-to-vehicle distance, r, and inertial path angle, γ, are supplied to the equations. The result is a set of parameters characterizing the conic two-body solution. The equations and the subroutine are described in Ref. 4.

The output from this computation is described in detail in Sec. III-E. However, when the resulting conic is a hyperbola, additional computations are made using as input the quantities given in the conic. These are described by the equations that follow. The unit, $\vec{\zeta}$, vector oriented normal to the plane of the conic is

$$\vec{\zeta} = \frac{\vec{r} \times \vec{v}}{\cos \gamma} \tag{85}$$

The auxiliary, $\vec{m}$, unit vector is constructed normal to $\vec{\zeta}$ and $\vec{r}$ by

$$\vec{m} = \frac{\vec{\zeta} \times \vec{r}}{|\vec{r}|} \tag{86}$$

Two additional unit vectors, $\vec{\xi}$ and $\vec{\eta}$, may be found by rotating $\vec{r}$ and $\vec{m}$ and $\vec{\zeta}$ through an angle $\hat{\circledS}$, the true anomaly. The $\vec{\xi}$ and $\vec{\eta}$ vectors are such that $\vec{\xi}$ is directed toward the perigee point while $\vec{\eta}$ is perpendicular to $\vec{\xi}$ and in the conic plane.

$$\left.\begin{aligned} \vec{\xi} &= \frac{\vec{r}}{|\vec{r}|} \cos \hat{\circledS} - \vec{m} \sin \hat{\circledS} \\ \vec{\eta} &= \frac{\vec{r}}{|\vec{r}|} \sin \hat{\circledS} + \vec{m} \cos \hat{\circledS} \end{aligned}\right\} \tag{87}$$

Now the unit $\vec{S}$ vector may be defined that is directed along the outgoing asymptote and the unit $\vec{b}$ vector normal to $\vec{S}$ and in the conic plane.

$$\left.\begin{aligned} \vec{S} &= \vec{\eta} \sin v_m + \vec{\xi} \cos v_m \\ \vec{b} &= -\vec{\eta} \cos v_m + \vec{\xi} \sin v_m \end{aligned}\right\} \tag{88}$$

where v_m is the maximum true anomaly.

These vectors may also be given in the space fixed coordinate system via the transformation matrix, $|C|$, given in Eq. (63).

III. PROGRAM OPERATION

A. General Description

The Powered Flight Trajectory Program, GNG06, is written in the SAP language for use with an IBM 704 with a 32,768 word storage. It directly occupies 11224 locations. The program requires no tape units other than for output option. The program may be written on tape and executed from logical tape unit No. 8. Writing the program on logical tape No. 8 is accomplished by replacing the binary transfer card of the binary deck by one which transfers to the octal location of the symbol TAPE. The program may be easily revised to be executed from IBM 704 with 8k storage and 4k drum storage. It utilizes standard SHARE subroutines wherever possible.

B. Program Control

The program utilizes the JP DEQ (5) subroutine as a differential-equation solver. This routine encompasses a Runge-Kutta 4th-order integration routine and independent and dependent variable control options. That is, control of the solution of differential equations is maintained by specifying the conditions under which discontinuities in the solutions must occur. Hence, changes in the forms of the equations occur when specific values of independent and/or dependent values are achieved. DEQ, then, integrates until the specified value of the variable is attained. Then control is transferred to the particular routine which carries out the change in the differential equations. When the change is accomplished, the derivatives are recomputed and control is returned to the control routine, DEQ.

The controls provided to DEQ, called triggers, are of either of two general types: (1) dependent variable triggers, or (2) independent variable triggers. Those controls in type (1) are automatically set by the program. Their sequence of use is initiated by input data that is described later (see Sec. III-D), and require no additional care.

Under the classification of controls of type (2) above, all except one is automatically set by the program. That is, the normal input data are used by the program to initiate the use of the triggers. The one exception is a generalized independent variable trigger, the use of which is left to the program user. That is, the user must provide input, in the proper format to this trigger. It is designed to allow changes in logic that are executed at specified values of the independent variable, time. For example, the selection of any printout interval may be made by this control at any time during the computation. The detailed description and use of these independent variable controls is given in Sec. III-D(4).

In general then, the program is designed to provide automatic control over the trajectory computation on the basis of dependent and independent variable controls that are automatically set on the basis of input performance data and on the basis of user-set independent variable controls.

C. Option Control

The various options available in the Powered Flight Trajectory Program were originally designed to be controlled by sense switches. However, all controls of option no longer require sense switches, but instead require control words that are read in as input data. The term sense switch is still used, but its meaning implies a control word. (see Input Format.) The following convention is used:

Control Word	Sense Switch Analogy
$n = 0$	up
$n \neq 0$	down

Resulting output, for example, is on-line or off-line as desired by one of these pseudo-sense switches. All of the options will be described in detail in the input description. (see Input Format.)

D. Input Format

Input to the Powered Flight Trajectory Program is accomplished with a modified version of the SHARE NYINP1 subroutine where all input is from cards. The routine requires input cards in the SAP format with certain symbols. Only two operation symbols are needed: DEC and TRA. The following conventions are standard:

$$\alpha \text{ DEC } x_1 x_2 x_3 \cdot x_4 x_5 \qquad\qquad \alpha \text{ DEC } x_1 x_2 x_3 x_4 x_5 E3$$

both produce a floating-point number, $x_1 x_2 x_3 \cdot x_4 x_5$, in the decimal location α.

$$\beta \text{ DEC } x_1 x_2 x_3 x_4 x_5$$

produces a fixed point number, $x_1 x_2 x_3 x_4 x_5$, in the decimal location β.

All input data are in floating-point form except where specified. Furthermore, there is an option available for units used, the control of which is described later. The units available are as follows:

Option 1: feet, radians, pounds, seconds

Option 2: meters, degrees, pounds, seconds

The selection of one of the above options implies that all input will be in these units with a few exceptions that will be well-defined. Furthermore, as will be shown later, output units will be compatible with these input units.

The program is designed to save entirely all input quantities. Thus, if repeat trajectories are computed with changes in input from an initial trajectory, then only the changes must be introduced in the subsequent trajectories. If memory is entirely cleared before introducing the program to the computer, then zero quantities need not be put in. Furthermore, options involving input need not be zero if the option is not used in a trajectory computation.

The input quantities will now be described in detail. In this description, the input quantities will be given with their program decimal location along with a brief definition of the quantity where it is needed. Also, units will be given if they are exceptions to the two-unit options given previously.

1. General Input

Location	Quantities	Description
350	I.D.	Identification number of form xx.xxx
351	Date	Identification date run of form xx.xx
352	Integer j	Selects jth performance data set as first set to be used
353	Search option	(Fixed point) 0 => no search 1 => univariate search 2 => bivariate search
354	Initial time	
355	Initial height	
356	Geodedic latitude	See Sec. III-C; equations
357	Longitude	See Sec. III-C; equations
358, 500	Azimuth	See Sec. III-C; equations

Location	Quantities	Description
390	Month	(Fixed point) Calendar date of launch (Used to produce input to Eq. 8.)
391	Day	
392	Year	
393	Launch time	Time (sec) referenced to midnight of above date
360	$v(0)$	Initial velocity and pitch angle if option 1 of Initial Conditions is used
361	$\chi(0)$	
363	$x_p(t_0)$	Position and velocity components if option 2 of Initial Conditions is used
364	$y_p(t_0$	
365	$z_p(t_0)$	
366	$\dot{x}_p(t_0)$	
367	$\dot{y}_p(t_0)$	
368	$\dot{z}_p(t_0)$	
503	$r(t_0)$	Polar coordinates and velocity if option 3 of Initial Conditions is used
523	$\psi(t_0)$	
543	$v_e(t_0)$	
563	⊙ (t_0)	
583	$\sigma(t_0)$	
603	$\lambda(t_0)$	
410	Sense switch 1	Not used by GNG06
411	Sense switch 2	= 0 => offline; ≠ 0 => online
412	Sense switch 3	Not used by GNG06
413	Sense switch 4	= 0 => not option 3; ≠ 0 => option 3
414	Sense switch 5	= 0 => not option 2; ≠ 0 => option 2
415	Sense switch 6	= 0 => ft, rad, lb, sec input-output ≠ 0 => meters, deg, lb, sec input-output

Performance data for motor-stage

230	F_0	Vacuum thrust (lb) If $F_0 = -1$ (Fixed point), use thrust from first of two polynomials available. If $F_0 = -2$ (Fixed point) use thrust from second of two polynomials available

Location	Quantities	Description
231	f_e	Exhaust area (ft^2)
232	W_g	Gross weight (lb). If = 0, compute gross weight from previous stage. (see Eq. 29b.)
233	W_e	Discard weight at end of stage
234	$\dot{W}_p$	Mass flow (lb sec). Same options as on F_0 above
235	W_f	Fuel weight shutoff (see Eq. 30 and 1b)
236	D	Effective diameter (ft)
237	C_{di}	Drag coefficient identifications (see Eq. 31-32)
238	Δt_b	Burning period if ≥ 0 Burning period determined by shutoff Eq. (64)-(67) if - 0
239	GDB	(Fixed point) Code word that determines path control option during burning
240	Δt_c	Coast period following burning portion of stage
241	GDC	(Fixed point) Code word that determines path control Control option during coast period[3]
242-301	Space provided for five additional sets of performance data identical with those described above	
310-323	Space provided for two F_0, 6th-degree polynomials for vacuum thrust option	
330-343	Space provided for two $\dot{W}_p$, 6th-degree polynomials for mass flow options	
435	m_1	Mach number bounds for drag coefficient polynomials (see Eq. 32)
436	m_2	
437	m_3	
438	m_4	
439-458	Space provided for five drag coefficient polynomials (see Eq. 32)	
459	m_1'	Mach number bounds for normal force coefficient polynomials (see Eq. 56)
460	m_2'	
461	m_3'	
462	m_4'	

[3] If GDB or GDC is set equal to zero, path control will not be changed from that option used until this time.

Location	Quantities	Description
463 - 482	Space provided for five normal force coefficient polynomials (see Eq. 56)	
370 - 373	μ_i, $i = 1, 2, 3, 4$	Four values of μ (Eq. 53) may be used in a trajectory
378 - 381	$\Delta\tau_i$, $i = 1, 2, 3, 4$	Four values of $\Delta\tau$ (Eq. 52) may be used in a trajectory
382 - 385	$\bar{\chi}_i$, $i = 1, 2, 3, 4$	Four values of $\bar{\chi}$ (Eq. 46) may be used in a trajectory
386 - 388	$\Delta\chi_i$, $i = 1, 2, 3$	Three values of $\Delta\chi$ (Eq. 48) may be used in a trajectory
485 - 491	Space provided for 6th-degree polynomial, $\alpha_p(t)$ (Eq. 41)	
492 - 498	Space provided for 6th-degree polynomial, $\alpha_y(t)$ (Eq. 42)	
700 - 741	Space provided for six 6th-degree polynomials $\chi(t)$, one polynomial per stage (Eq. 47)	
750 - 791	Space provided for six 6th-degree polynomials $\tau(t)$, one polynomial per stage (Eq. 51)	

2. Station Coordinate Computation Inputs

650	ϕ_i	Geocentric latitude of station
651	$\odot_i$	Longitude of station
652	R_i	Earth-center-to-station distance
653	F_i	Frequency and constants of Eq. (84)
654	A_i	Frequency and constants of Eq. (84)
655	B_i	Frequency and constants of Eq. (84)
656	C_i	Frequency and constants of Eq. (84)
657	D_i	Frequency and constants of Eq. (84)
658 - 690	Space provided for four more sets of station data	

3. Shutoff Equation (Eq. 66)

50	k_1	Multiplier coefficient
51	k_2	Multiplier coefficient
52	k_3	Multiplier coefficient
53	k_4	Multiplier coefficient
54	a'	Drift coefficient
55	b'	Drift coefficient
56	$V_x(t_0)$	Initial measurable velocity
57	V_{xs}	Standard shutoff velocity

4. Independent Variable Control Input

The following is a list of control numbers that will bring about changes of logic or changes of control in the program. These control numbers when provided as shown below with a specific value of the independent variable, time, will bring about the desired change in logic or control when this value of time is reached.

Control Number (Fixed Point)	Function
100	Dummy No. => error
101	Set Δt = 1 sec
102	Set Δt = 2 sec
103	Set Δt = 10 sec
104	Set Δt = 50 sec
105	Set print interval = 0.5 sec
106	Set print interval = 1.0 sec
107	Set print interval = 2 sec
108	Set print interval = 10 sec
109	Set print interval = 100 sec
110	No regular printout
111	Unconditional printout
112	Printout and halt
113	Printout, reset program, and restart
Path control selection	
114	Pitch with constant attack angle
115	Pitch with attack angle polynomial
116	Zero lift
117	Gravity turn
118	Vertical flight

Control Number (Fixed Point)	Function
119	Constant χ_i, $i = 1, 2, 3, 4$ (i progresses with each use of this control)
120	$\chi(t)$ polynomial (automatically uses that polynomial associated with current stage)
121	Modify pitch angle by $\Delta\chi_i$ and hold constant attitude (i progresses automatically with each use of this control)
122	Let $\tau = 0$
123	No yaw restriction
124	$\tau(t)$ polynomial (automatically uses that polynomial associated with current stage)
125	Modify yaw angle by $\Delta\tau_i$ and hold constant attitude (i progresses automatically with each use of this control)
126	Yaw with attack angle polynomial
138	Reference pitch angle to local horizon through angle μ_i (i progresses automatically with each use of this control)
128	Set next stage
129	Coast
130	Transfer control to DBH06. (Assumes that DBH06 is next record on tape No. 8)
133	Print two-body solution (conic)
134	Print two-body solution with regular print
135	Begin station printout
136	Stop station printout (automatically set at beginning of trajectory computation)
137	End univariate and bivariate search

Using the above control numbers, selection of a particular function to be carried out at a particular time, t_s, is accomplished by input of information in the following card format:

Location	Time	Flight (Control No.)
150	t_{s1},	xxx
152	t_{s2},	xxx
154	t_{s3},	xxx

There is a maximum of 25 such controls. It is pointed out at this time that the above given control numbers are those that must be provided in the performance data input at GDB and GDC. It is also pointed out that the control words 128 and 129 are set by the program and need not, except for special applications, be set as an independent variable control. Control numbers 101 - 104 need not be set except for special applications since the program will use $\Delta t = 2$ sec during powered flight and $\Delta t = 10$ sec during coast. Control numbers 112 - 113 must be used to terminate the computation. Output automatically occurs at the beginning of computation and at stage changes and coast, and at the end of the trajectory computation. Hence, control number 110 will give no additional output. However, control numbers 105 - 109 will, in addition to the automatic printout, give printout at the prescribed printout intervals. Control number 111 produces one printout at the prescribed time. If the printout interval is less than the integration interval, the program is forced to use an integration interval smaller than that desired in order to reach each print time.

5. Search Input

A generalized univariate and bivariate search routine is included so that searches may be made for desired values of selected variables by varying other selected variables. This differential correction scheme requires the selection of 'independent' search variables and 'dependent' desired variables. The search routine, then, using the Powered Flight Trajectory Program as a subroutine, varies the values of the 'independent' variables in a manner such as to converge on the desired values of the 'dependent' variables at the end of the trajectory.

The program uses standard SHARE subroutines JP TARN (Univariate Search) and JP WEIR (Bivariate Search) along with JP GNAT (Lagrange's Interpolation Routine). The program allows the selection of any input quantities and any computed quantities for independent and dependent variables. In addition, on input, the desired values of the dependent variables and allowable errors in these values are provided. Also, nominal increment values of the independent variables are entered. In operation, the program computes a trajectory using the initial or nominal values of the independent variables; I_i. It then computes trajectories using values of I_i modified by the increments, ΔI_i, one trajectory per change of variable. Using the differences, ΔD_i, in the computed values of the dependent variables, D_i, from their values from the nominal trajectory, the search routine computes a new set of ΔI_i, and the procedure is repeated. If the procedure converges, the computation will terminate with the computed values of the D_i equal to, within the allowable error, the desired value of D_i. At this time, of course, the values of the I_i are such as to bring about the originally desired values of the I_i.

Selection of no search, univariate, or bivariate search was indicated in the search option at location 353 in the input quantities. The input format of quantities required for these searches is given as follows:

Location	Quantity	Description
70	$L\,(V_{r_{(1)}})$	Decimal location of independent variables
71	$L\,(V_{r_{(2)}})$	
82	δ_1	Decrements of independent variables
83	δ_2	
76	$L\,(V_{D_{(1)}})$	Decimal location of dependent variables
77	$L\,(V_{D_{(2)}})$	
78	$D_{D_{(1)}}$	Desired value of dependent variables
79	$D_{D_{(2)}}$	
86	$E_{(1)}$	Allowable errors in value of dependent variables
87	$E_{(2)}$	

If univariate search is used, then the first of the above pairs of quantities need be given, while if bivariate search is used, all of the above information is required. The final input card must return control to the input subroutine. This card must be TRA 3,4.

E. Output Format

The output, as explained earlier, is given in either the units ft, rad, lb, sec, or meters, deg, lb, sec, and consists of three basic parts. These are the heading, the normal print, and the motor data print. The heading is given initially with every trajectory computation, and it includes pertinent initial data and identification. The normal print is given always at the beginning of the computation, at the beginning of all coast periods and stages, and at the termination of the computation. However, through the use of the aforementioned independent variable controls, additional normal print may be given at regular intervals. The motor data print are given at the initiation of a new power stage. These three parts will now be described in the following tabulation.

Heading

Symbol	Description
IDENT	Identification number
DATE	Date of trajectory computation
AZI	Azimuth of launch
LAT	Geocentric latitude of launch
GED	Geodedic latitude of launch
LØN	Longitude of launch
RAD	Earth-to-launch distance
LAUNCH DATE	Calendar date of launch
JDT	Julien date of launch
GHA	Greenwich hour angle at date
TL	Launch time referenced to midnight of above date

Normal Print

Symbol	Description
TIM	Time
ACX	Measurable acceleration along $\vec{c}$
INA	Measurable velocity along $\vec{c}$
DDX DDY DDZ	Acceleration components in inertial coordinate system

Symbol	Description
DT	Sublime integration interval
DTT	Integration interval
XP YP ZP	Position components in inertial coordinate system
DXP DYP DZP	Velocity components in inertial coordinate system
V	Inertial velocity
GAM	Inertial path angle
XM YM ZM	Measurable position components in inertial coordinate system
DXM DYM DZM	Measurable velocity components in inertial coordinate system
VE	Earth-fixed velocity
PTH	Earth-fixed path angle
ARC	Angle at Earth center from current to launch position
ALT	Height
R	Earth-center-to-vehicle distance
XDD YDD ZDD	Measurable acceleration components in inertial coordinate system
LAT	Geocentric latitude
LØN	Longitude
M	Mass (lb)
F	Thrust
A	Axial drag force
N	Normal force
CHI	Inertial vehicle pitch angle
TAU	Inertial vehicle yaw angle
SIG	Earth-fixed path azimuth
SGI	Inertial path azimuth

Symbol	Description
XA YA ZA	Position components in space fixed coordinate system
DXA DYA DZA	Velocity components in space fixed coordinate system
ALA	Attack angle
PAY	Payload weight

Motor Data Print

Symbol	Description
FRC	Vacuum thrust
FE	Exhaust area
WG	Gross weight
WPD	Mass flow (lb)
DIA	Diameter (ft)
CD	Drag coefficient
BRN	Burning period (sec)
CST	Coast period (sec)

When control is used to compute the station observations, a printout is given by the program describing the data at each station required. The symbols and quantities are given as follows:

Symbol	Description
LAT	Geocentric latitude of station
LØN	Longitude of station
RAD	Slant range to vehicle
ELV	Elevation angle of vehicle
DCL	Declination of vehicle
SIG	Azimuth direction of vehicle
HAN	Hour angle of vehicle
RDR	Slant range rate

Symbol	Description
ELR	Elevation angle rate
DCR	Declination rate
SGR	Azimuth rate
HAR	Hour angle rate
RDD	Slant range rate change
LØK	Look angle
PØL	Polarization angle
FRQ	Frequency received at station

These quantities are repeated for each station.

When control is used to compute the two-body conic, a printout is given by the program describing the resulting conic path and its parameters. While it was pointed out that all units of input and output are consistent with the two options described earlier, one major exception exists in the conic printout. These are demonstrated.

Input Units	Conic Print Units
ft, rad, sec, lb	miles, rad, sec, lb
meters, deg, sec, lb	meters, deg, sec, lb

The conic printout format is now given.

Conic-Ellipse	Description
MØM	Momentum
ENR	Energy
ECC	Eccentricity
AXS	Semi-major axis
PER	Period (days)
ANM	True anomaly
APØ	Apogee distance
PGE	Perigee distance
VAP	Velocity at apogee

Conic-Ellipse	Description
VPG	Velocity at perigee
TPG	Time to perigee
EAN	Eccentric anomaly

Conic-Hyperbola	Description
MØM	Momentum
ENR	Energy
ECC	Eccentricity
AXS	Semi-transverse axis
PDT	Perpendicular distance
ANM	True anomaly
MAX	Maximum true anomaly
PGE	Perigee distance
EXV	Excess hyperbolic velocity
VPG	Velocity at perigee
TPG	Time to perigee
XI	$\vec{\xi}$ vector components
ETA	$\vec{\eta}$ vector components
ZETA	$\vec{\zeta}$ vector components
S	$\vec{S}$ vector components
M	$\vec{m}$ vector components
B	$\vec{b}$ vector components

The latter six vectors are repeated in the space fixed coordinate system.

F. Timing

The time required to compute a complete trajectory may be estimated from the number of integration steps taken by the Runge-Kutta integration routine. This may be done easily with knowledge of the interval size. As was pointed out in Sec. III-D(4), integration intervals of 2 sec and 10 sec are automatically used during burning and coasting periods, respectively. These may be used to determine the total number of integration steps. The following formula is given for approximating the computing time:

$$t = \frac{n}{50}$$

where n = number of integration steps, and t is in minutes.

Example: The time of computing will be found for the test case in Appendix IV.

	Burning Time	Δt	Integration Steps
Stage 1	171.11	2	86
Stage 2	72.0	2	36
Coast 1	15.0	10	2
Coast 2	1.89	10	1
		Total Steps	125

$$t = \frac{125}{50} = 2.5 \text{ min}$$

APPENDIX I. Glossary of Equation Notation and Program Symbols[1]

Equation	Program	Definition
a	MAJAX	Earth semi-major axis
b	MINAX	Earth semi-minor axis
e	ECCØN	Earth eccentricity
ω	ØMEGA	Earth angular rotation
t_0	TZ	Initial time
Ψ_0'	PSIZP	Initial geodetic latitude
λ_0	LAMZE	Initial longitude
σ_L	AZIM	Initial azimuth
h_0	HITEZ	Initial height of coordinate system
Ψ_0	PSIZ	Initial geocentric latitude
r_s	RS	Radius of Earth
r_0	RZERØ	Initial Earth-center-to-vehicle distance
β_0	BETAZ	Geodetic-geocentric latitude difference
$\sin k$	SINK	Sine of west azimuth
$\cos k$	CØSK	Cosine of west azimuth
$x_p''(0)$	XPRIMZ	Coordinates of the origin of the inertial Earth-centered coordinate system
$y_p''(0)$	YPRIMZ	
$z_p''(0)$	ZPRIMZ	
Ω_1'	ØMGP1	Components of unit Earth spin axis vector
Ω_2'	ØMGP2	
Ω_3'	ØMGP3	
Ω_1	ØMEG1	Components of Earth spin axis vector
Ω_2	ØMEG2	
Ω_3	ØMEG3	

[1] Program symbols refer to working storage location and not necessarily that location in which a quantity is initially stored on input.

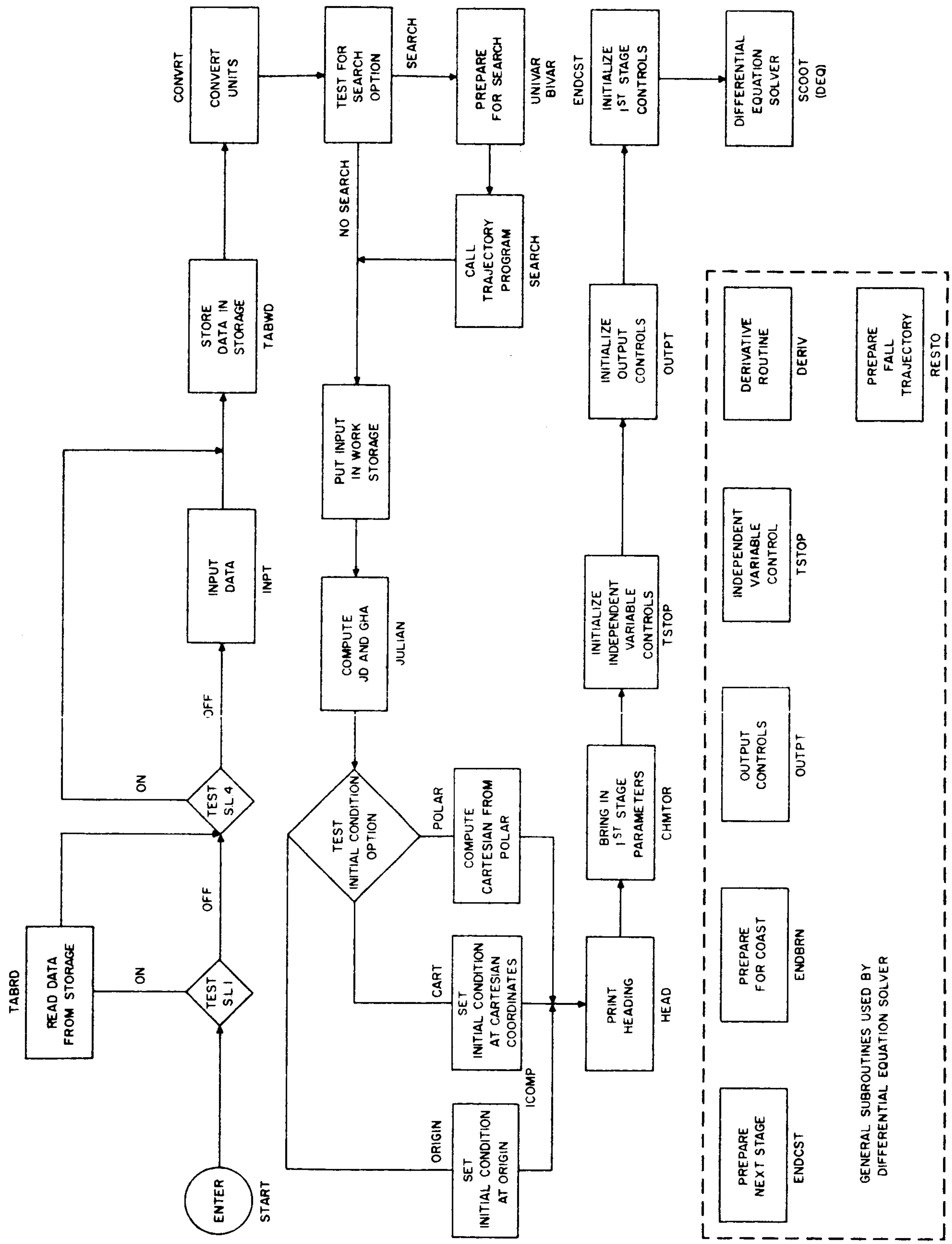
Equation	Program	Definition
$\mathrm{GHA}(t_m)$	GHA	Greenwich hour angle at midnight of date
JD	JULIE	Julian date
t_L	TLACH	Launch time referenced to midnight
Θ_L	ASCL	Origin right ascension at launch time
$\mathrm{GHA}(t_0)$	GHAT	Greenwich hour angle at launch time
Θ	RTASN	Vehicle right ascension at time
X_L	CAPXL	
Y_L	CAPYL	
Z_L	CAPZL	Position and velocity components at launch in space-fixed coordinate system
$\dot{X}_L$	CAPXLD	
$\dot{Y}_L$	CAPYLD	
$\dot{Z}_L$	CAPZLD	
X	CAPX	
Y	CAPY	
Z	CAPZ	Position and velocity components of time in space-fixed coordinate system
$\dot{X}$	CAPXD	
$\dot{Y}$	CAPYD	
$\dot{Z}$	CAPZD	
$\ddot{x}_p$	XDDØT	
$\ddot{y}_p$	YDDØT	
$\ddot{z}_p$	ZDDØT	
$\dot{x}_p$	XDØT	
$\dot{y}_p$	YDØT	Acceleration, velocity, and position components in the launch-centered inertial coordinate system
$\dot{z}_p$	ZDØT	
x_p	LØCX	
y_p	LØCY	
z_p	LØCZ	
x''_p	XPRIM	
y''_p	YPRIM	Position coordinates in Earth-centered inertial coordinate system
z''_p	ZPRIM	
r	RADIS	Earth-center-to-vehicle distance

Equation	Program	Definition
r_1'	AR1	Components of unit $\vec{r}$ vector
r_2'	AR2	
r_3'	AR3	
Ψ	PSI	Geocentric latitude
A'	APRIM	Components of gravity potential equation
B'	BPRIM	
g_1	GRV1	Components of gravity in inertial coordinate system
g_2	GRV2	
g_3	GRV3	
V_{1p}	V1P	Components of Earth-fixed velocity in inertial coordinate system
V_{2p}	V2P	
V_{3p}	V3P	
v_e	VR	Earth-fixed velocity
v_1'	VEL1	Components of unit velocity vector in inertial coordinate system
v_2'	VEL2	
v_3'	VEL3	
h	HITE	Height above oblate-spheroidal Earth
$p(h)/p(0)$	PRERT	Atmosphere pressure ratio
$p(h)$	PRESS	Atmospheric pressure
$p(0)$	SLPRS	Sea level atmospheric pressure
$a(h)$	ACVEL	Accoustic velocity
F_0	FZERØ	Vacuum thrust (lb)
f_e	FE	Exhaust area (ft^2)
W_g	WG	Gross weight (lb)
W_e	WEMPT	Stage empty weight
W_f	WFUEL	Fuel weight at shutoff
$\dot{W}_p$	WPDØT	Mass flow per unit time (lb/sec)
F	FØRCE	Thrust

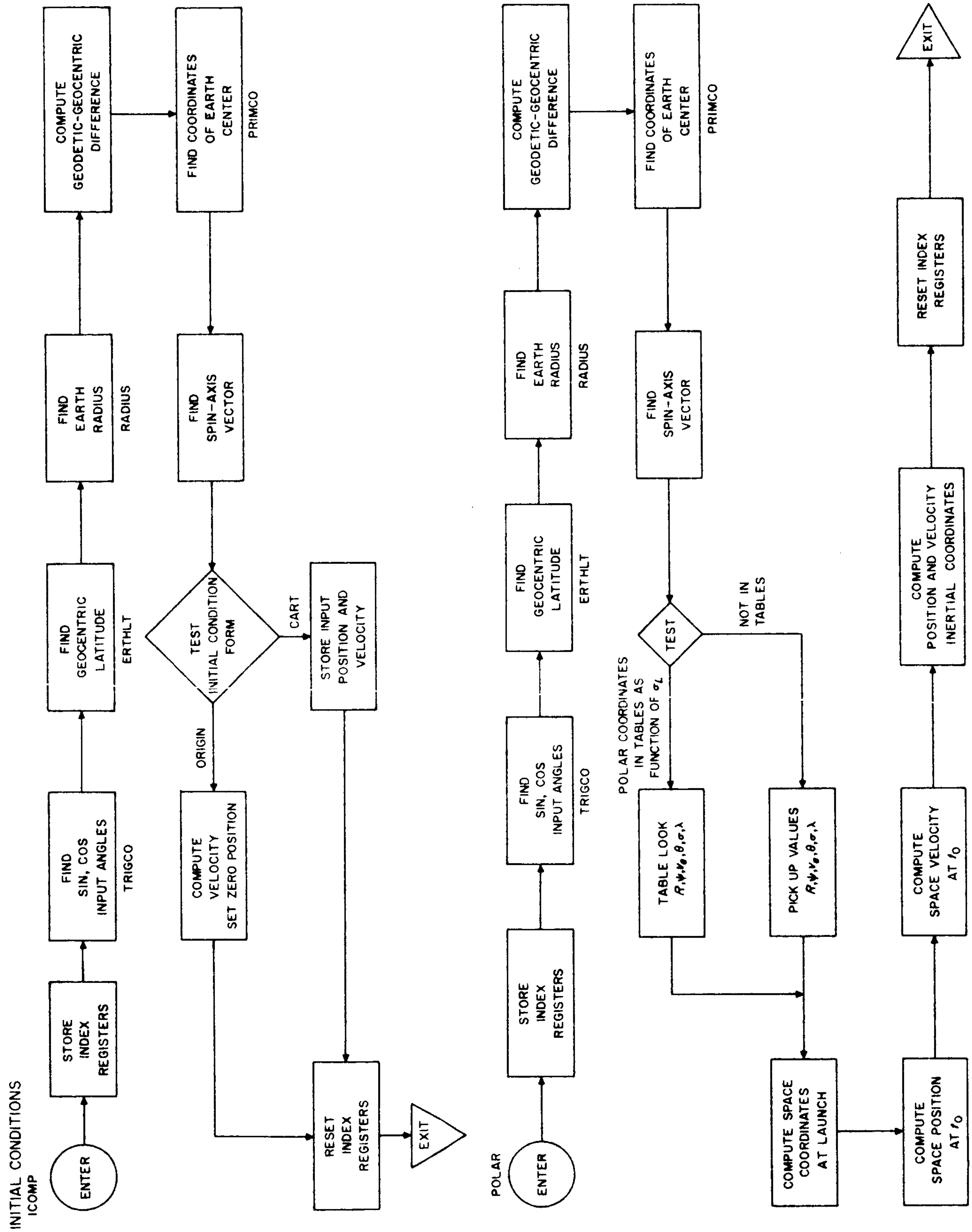
Equation	Program	Definition
$m(t)$	MASS	Mass (slugs)
g_0	GZERO	Mass conversion unit
c_{d_0}	CDØ	Drag coefficient
M	MACH	Mach number
q	QU	Dynamic pressure
d	DIAM	Vehicle effective diameter
Θ	THETA	Earth-fixed path angle
γ	GAMMA	Inertial path angle
v_i	SPEED	Inertial velocity
σ	SIGMA	Earth-fixed path azimuth angle
σ_i	SIGMAI	Inertial path azimuth angle
χ	CHI	$\vec{c}$ pitch angle
τ	TAU	$\vec{c}$ yaw angle
c_1 c_2 c_3	CE1 CE2 CE3	Components of unit $\vec{c}$ vector in inertial coordinate system
α_p	ALFM	Pitch angle of attack
α_y	ALFN	Yaw angle of attack
α	ALPHA	Total angle of attack
n_1 n_2 n_3	EN1 EN2 EN3	Components of unit normal force vector in inertial coordinate system
C_z'	PARCN	Normal force coefficient
N	NØRM	Total normal force
ϕ	PHI	Angle subtended at the Earth center from origin to current position of vehicle
$\Delta\lambda$	DELAM	Change in longitude

Equation	Program	Definition
λ	LAMBDA	Longitude
$\ddot{x}_m$	XSUBM	Measurable acceleration, velocity, and position coordinates in inertial coordinate system
$\ddot{y}_m$	YSUBM	
$\ddot{z}_m$	ZSUBM	
$\dot{x}_m$	DXM	
$\dot{y}_m$	DYM	
$\dot{z}_m$	DZM	
x_m	XM	
y_m	YM	
z_m	ZM	
a_x	ASUBX	Total measurable acceleration in $\vec{c}$ direction
V_x	VSUBX	Total measurable velocity in $\vec{c}$ direction
V_s	VSUBS	Measurable shutoff velocity
V_{x_s}	VXSTD	Standard measurable shutoff velocity
$V_x(t_0)$	VXTØ	Initial measurable velocity in $\vec{c}$ direction
R_s	RAD	Slant range, rate, and change of rate from ith station
$\dot{R}_s$	RDT	
$\ddot{R}_s$	RDDT	
α	HA	Hour angle and rate
$\dot{\alpha}$	HART	
δ	DCL	Declination and rate
$\dot{\delta}$	DCD	
e	ELEV	Elevation and rate
$\dot{e}$	ELEVD	
σ	SIG	Azimuth and rate from ith station
$\dot{\sigma}$	SIGD	
L	LCK	Look angle from ith station
p	PØL	Polarization angle
f_i	FRQ	Frequency

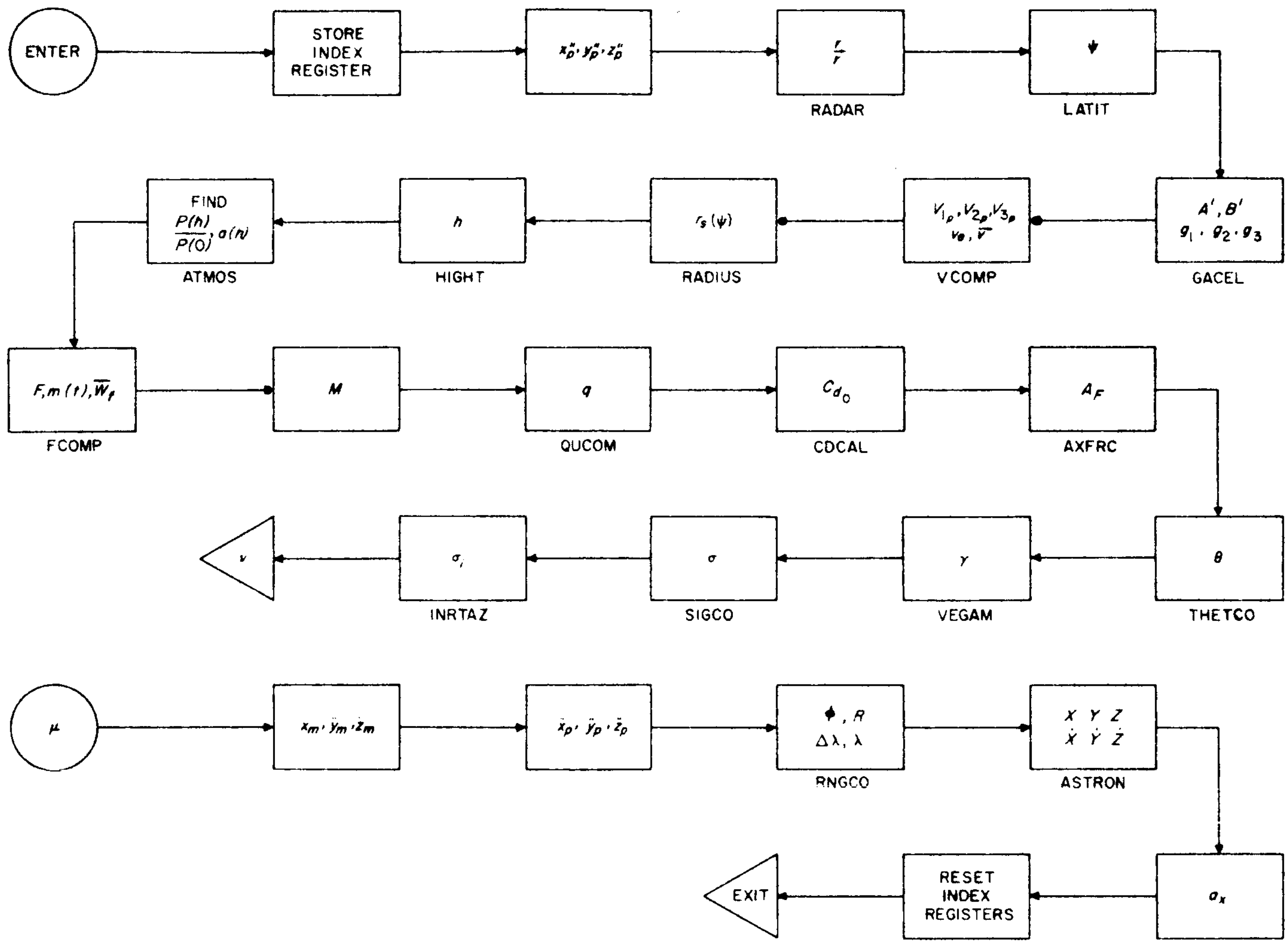
APPENDIX 2. FLOW CHARTS



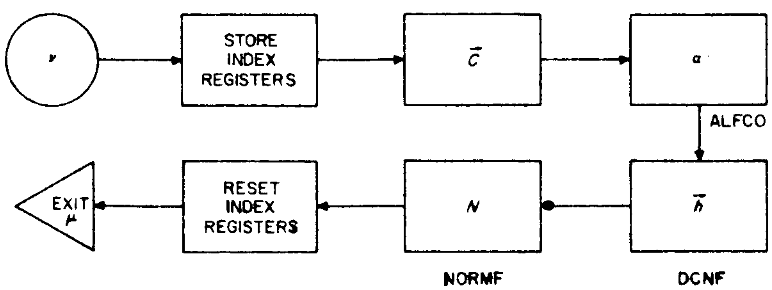
TRAJECTORY PROGRAM


INITIAL CONDITIONS
ICOMP
ENTER
STORE INDEX REGISTERS
FIND SIN, COS INPUT ANGLES
TRIGCO
FIND GEOCENTRIC LATITUDE
ERTHLT
FIND EARTH RADIUS
RADIUS
COMPUTE GEODETIC-GEOCENTRIC DIFFERENCE
FIND COORDINATES OF EARTH CENTER
PRIMCO
FIND SPIN-AXIS VECTOR
TEST INITIAL CONDITION FORM
ORIGIN
CART
COMPUTE VELOCITY SET ZERO POSITION
STORE INPUT POSITION AND VELOCITY
RESET INDEX REGISTERS
EXIT
POLAR
ENTER
STORE INDEX REGISTERS
FIND SIN, COS INPUT ANGLES
TRIGCO
FIND GEOCENTRIC LATITUDE
ERTHLT
FIND EARTH RADIUS
RADIUS
COMPUTE GEODETIC-GEOCENTRIC DIFFERENCE
FIND COORDINATES OF EARTH CENTER
PRIMCO
FIND SPIN-AXIS VECTOR
TEST
POLAR COORDINATES IN TABLES AS FUNCTION OF σ_L
NOT IN TABLES
TABLE LOOK $R, \psi, \nu_e, \theta, \sigma, \lambda$
PICK UP VALUES $R, \psi, \nu_e, \theta, \sigma, \lambda$
COMPUTE SPACE COORDINATES AT LAUNCH
COMPUTE SPACE POSITION AT t_0
COMPUTE SPACE VELOCITY AT t_0
COMPUTE POSITION AND VELOCITY INERTIAL COORDINATES
RESET INDEX REGISTERS
EXIT

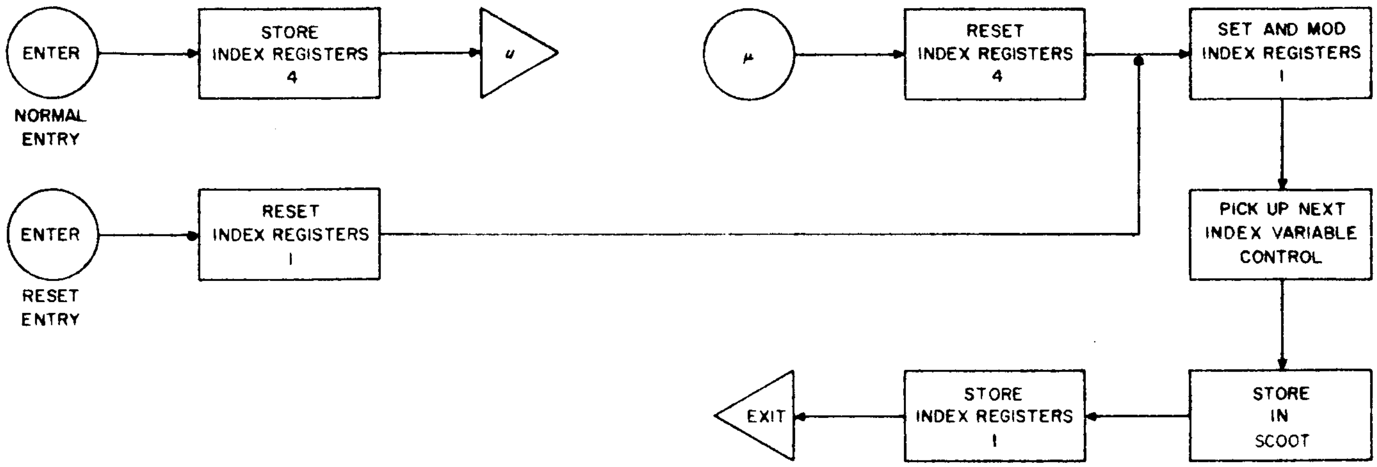
DERIVATIVE ROUTINE



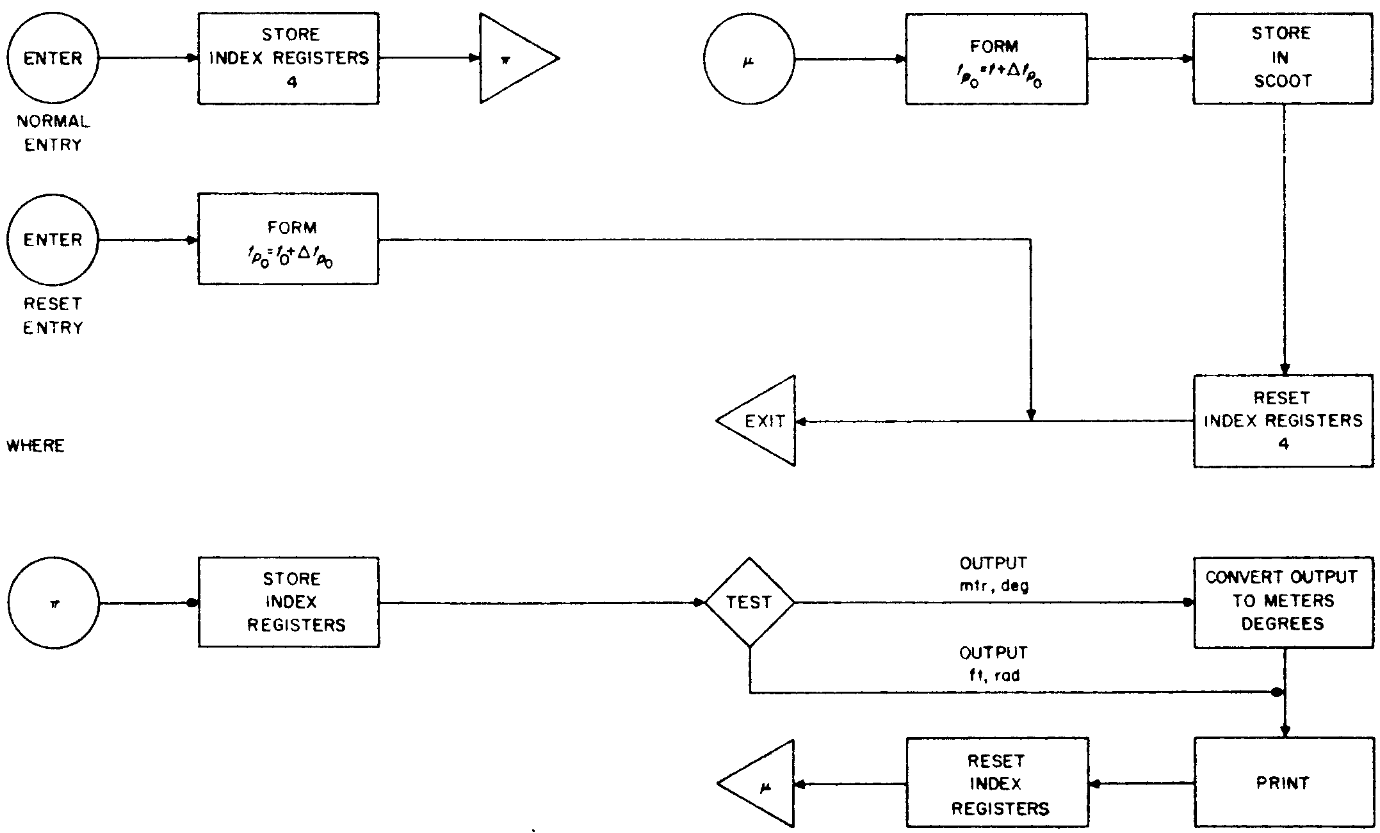
GENERAL FORM OF SELECTED PATH CONTROL ROUTINE (LAM i)



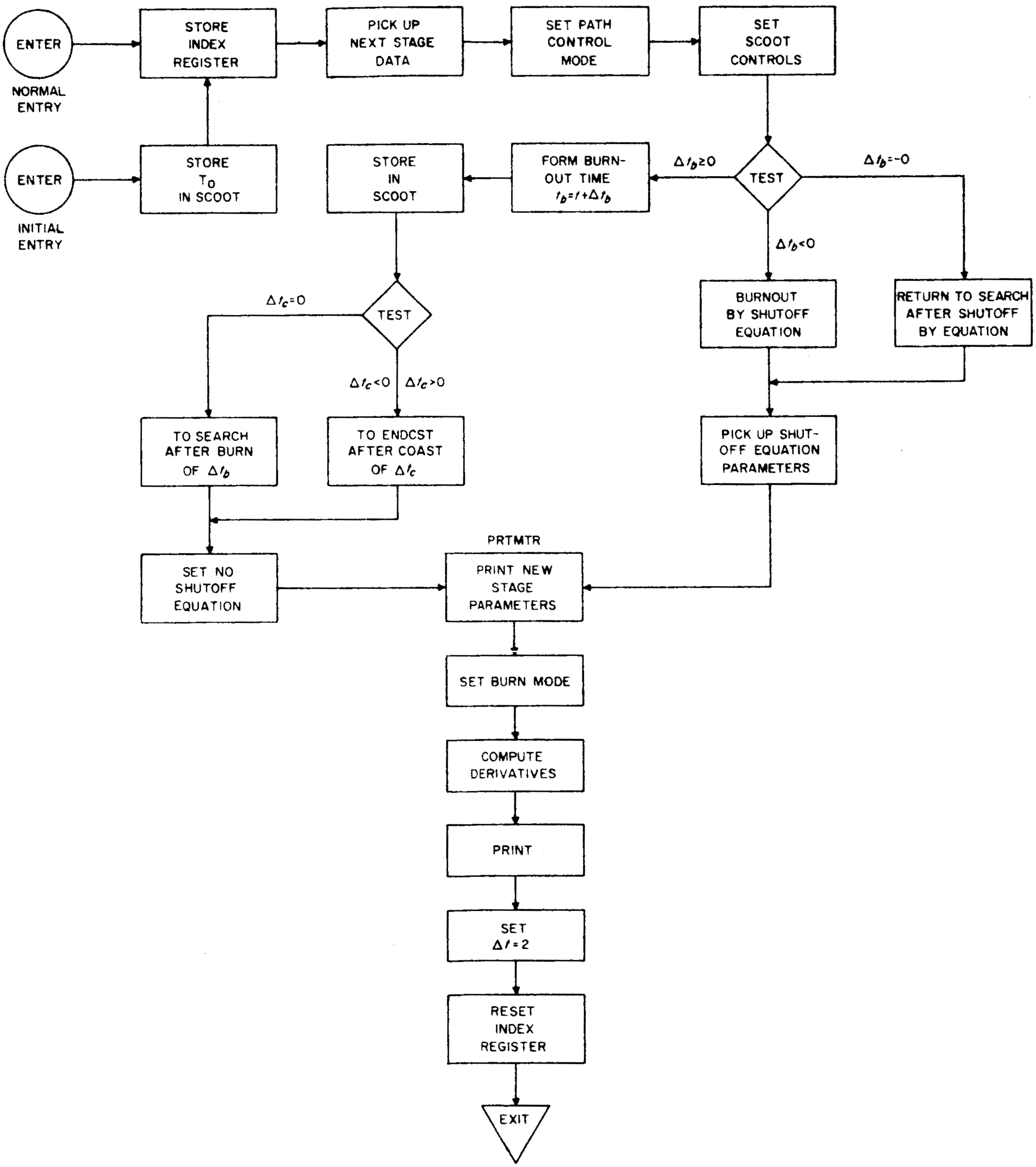
INDEPENDENT VARIABLE CONTROL ROUTINE



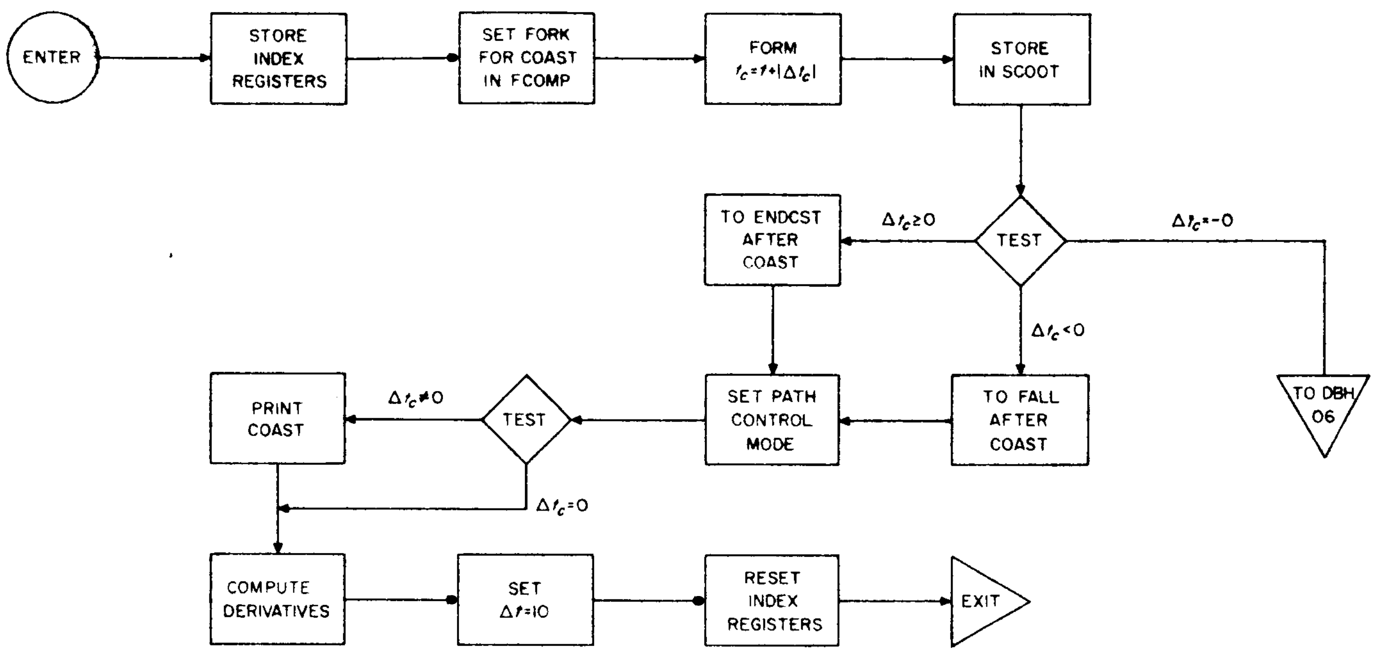
OUTPUT ROUTINE



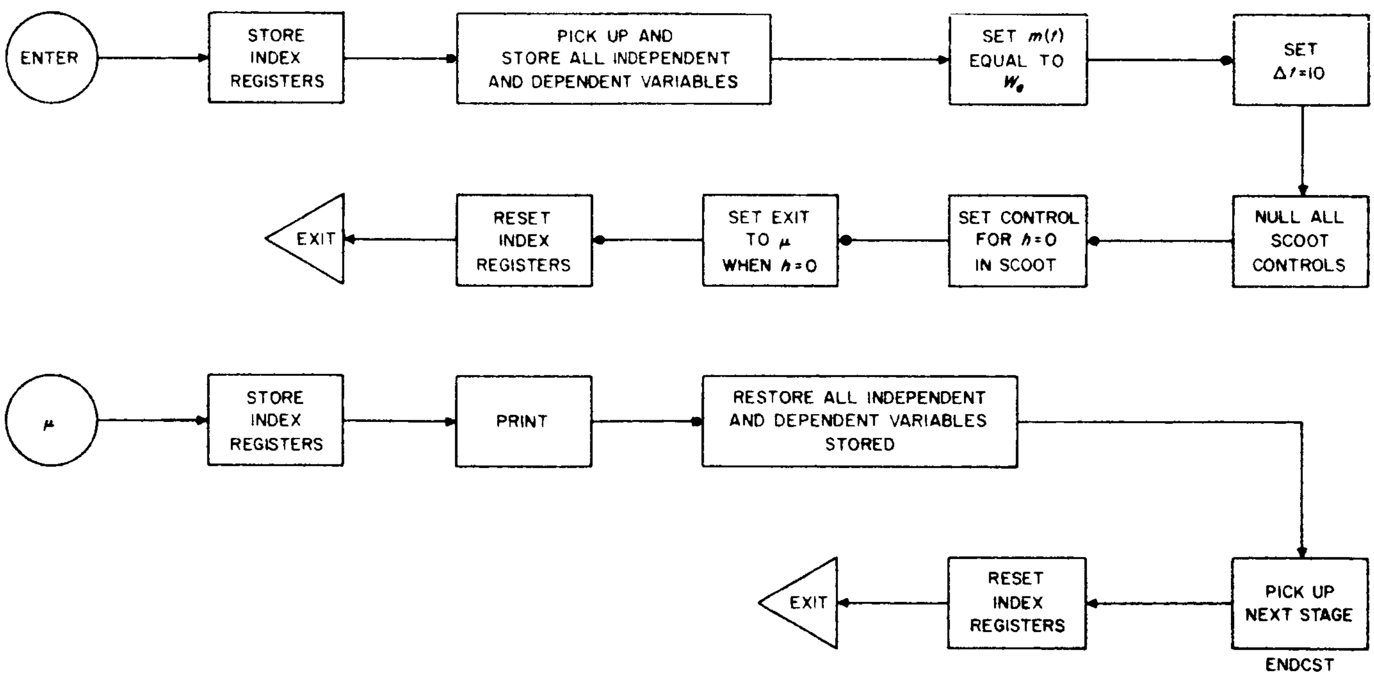
PREPARE NEXT STAGE



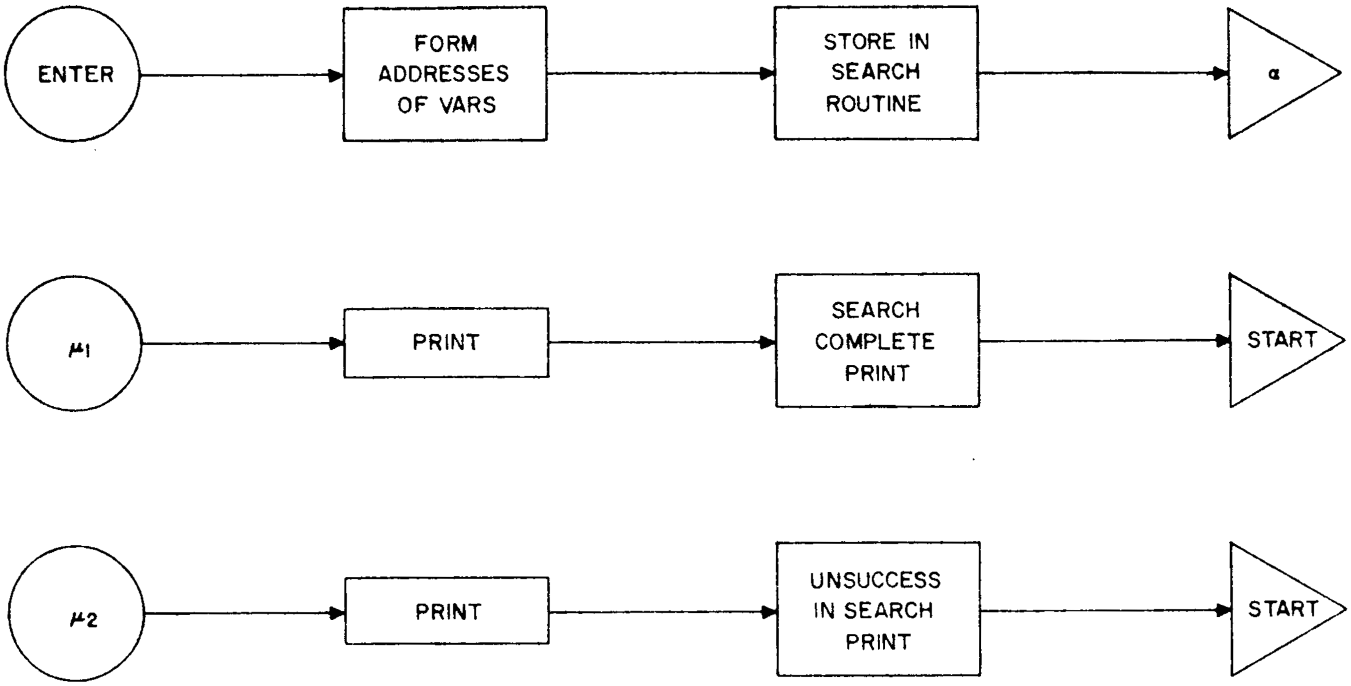
PREPARE COAST



PREPARE FALL TRAJECTORY



SEARCH
UNIVAR - BIVAR



SEARCH ROUTINE OF FOLLOWING FORM:

EACH TRAJECTORY PASS IS TERMINATED
BY ROUTINE OF FOLLOWING FORM:

APPENDIX III. Standard Atmosphere Data

ARDC STANDARD, ATMOSPHERE TABLE, 1957 fit to the following polynomials.

Pressure Ratio, $P(h)/P(0)$

Let $y_n = y/300000$ where y is altitude in ft, and let

$$Y = e^{\alpha y_n + \beta}$$

If

$$P_i(y) = a_0 + a_1 e^{\alpha y_n + \beta} + \cdots + a_0 e^{6(\alpha y_n + \beta)}$$

then

$$\frac{P(h)}{P(0)} = \begin{cases} P_1(y);\ -2000 \leq y < 53000,\ \alpha = -5.55555,\ \beta = -0.037033 \\ P_2(y);\ 53000 \leq y < 103000,\ \alpha = -6.122461,\ \beta = 1.081631 \\ P_3(y);\ 103000 \leq y < 161000,\ \alpha = -5.263157,\ \beta = 1.807015 \\ P_4(y);\ 161000 \leq y < 219000,\ \alpha = -10.169524,\ \beta = 5.437638 \\ P_5(y);\ 219000 \leq y < 266000,\ \alpha = -13.043504,\ \beta = 9.565234 \\ P_6(y);\ 266000 \leq y \leq 300000,\ \alpha = -18.181818,\ \beta = 16.181818 \end{cases}$$

and the coefficients of the $P_i(y)$ are as follows:

	$P_1(y)$	$P_2(y)$	$P_3(y)$
a_0	0.23789984×10^0	0.14458900×10^0	$-0.12415715 \times 10^{-1}$
a_1	-0.16838494×10^1	-0.15571538×10^1	0.12227733×10^0
a_2	0.40535928×10^1	0.69038915×10^1	-0.48021626×10^0
a_3	-0.20377407×10^0	-0.15657724×10^2	0.10065048×10^1
a_4	-0.42025751×10^1	0.19784885×10^2	-0.11347710×10^1
a_5	0.41834655×10^1	-0.12955566×10^2	0.66765279×10^0
a_6	-0.13098712×10^1	0.34371573×10^1	-0.15924085×10^0

	$P_4(y)$	$P_5(y)$	$P_6(y)$
a_0	$-0.56179751 \times 10^{-4}$	$0.23741591 \times 10^{-5}$	$-0.77198386 \times 10^{-6}$
a_1	$0.14188436 \times 10^{-2}$	$0.23035201 \times 10^{-4}$	$0.27363145 \times 10^{-4}$
a_2	$-0.32060724 \times 10^{-2}$	$0.35677239 \times 10^{-3}$	$0.22841154 \times 10^{-3}$
a_3	$0.11766024 \times 10^{-1}$	$-0.76502705 \times 10^{-3}$	$0.22841154 \times 10^{-3}$
a_4	$-0.21279992 \times 10^{-1}$	$0.80790626 \times 10^{-3}$	$-0.30004863 \times 10^{-3}$
a_5	$0.18579850 \times 10^{-1}$	$-0.36553984 \times 10^{-3}$	$0.19021516 \times 10^{-3}$
a_6	$-0.62353042 \times 10^{-2}$	$0.36964025 \times 10^{-4}$	$-0.45261983 \times 10^{-4}$

rms error:

$P_{1(y)}$: 2.223321×10^{-4}, $P_{3(y)}$: 2.873879×10^{-5}, $P_{5(y)}$: 2.808945×10^{-7}

$P_{2(y)}$: 2.846544×10^{-4}, $P_{4(y)}$: 1.9210138×10^{-6}, $P_{6(y)}$: 4.1468195×10^{-7}

Accoustic Velocity, $a(h)$

Let $y_n = y/300000$ where y is altitude in ft. If $a_i(y) = a_0 + a_1 y_n + \cdots + a_6 y_n^6$, then

$$a(y) = \begin{cases} a_1(y) & -2000 \leq y < 38000 \\ 968.08 & 38000 \leq y < 80000 \\ a_2(y) & 80000 \leq y < 160000 \\ 1105.7 & 160000 \leq y < 170000 \\ a_3(y) & 170000 \leq y < 250000 \\ 922.8 & 250000 \leq y \leq 300000 \end{cases}$$

and the coefficients of the $a_i(y)$ are as follows:

	$a_1(y)$	$a_2(y)$	$a_3(y)$
a_0	0.11161787×10^{4}	0.21401073×10^{4}	$-0.18103956 \times 10^{6}$
a_1	$-0.11675900 \times 10^{4}$	$-0.12858749 \times 10^{5}$	0.14797600×10^{7}
a_2	0.56561107×10^{4}	0.39502867×10^{5}	$-0.49804247 \times 10^{7}$
a_3	$-0.32687665 \times 10^{6}$	0.26060088×10^{5}	0.88937720×10^{7}
a_4	0.66136155×10^{7}	$-0.35596453 \times 10^{6}$	$-0.88885487 \times 10^{7}$

	$a_1(y)$	$a_2(y)$	$a_3(y)$
a_5	-0.58206908×10^8	0.66814040×10^6	0.47120469×10^7
a_6	0.18605109×10^7	-0.40833579×10^6	-0.10347544×10^7

rms error:

$a_1(y)$: 0.2644653

$a_2(y)$: 0.25603012

$a_3(y)$: 0.34602895

APPENDIX IV. Sample Trajectory Input and Output

POWERED FLIGHT TRAJECTORIES GNG06

EXAMPLE CASE

NAME JOHN DOE

DATE ______ EXT. ______

CH. NO. ______

LOC	OP			EXPLANATION
INITIAL CONDITIONS:				
.3.5.0	D.E.C	1.		ID NO. 1.000 xxx.xxx
.3.5.1	D.E.C	5.06		DATE 5/6 xx.xx
.3.5.2	D.E.C	1.0		INITIAL STAGE n = 1, 2, ..., 6
.3.5.3	D.E.C	0	Fx. Pt.	SEARCH OPTION 0 ⇒ NO SEARCH
. . .	. .			1 ⇒ UNIVARIATE
. . .	. .			2 ⇒ BIVARIATE
. . .	. .			3 ⇒ EXTENDED
LAUNCH SIGHT CONDITIONS:				
.3.5.4	D.E.C	150.0		INITIAL TIME
.3.5.8	D.E.C	55.		N. AZIMUTH
PAD =				
.3.5.5	D.E.C	0		INITIAL HEIGHT
.3.5.6	D.E.C	28.		GEODETIC LATITUDE
.3.5.7	D.E.C	279		LONGITUDE
. . .	. .			
.3.9.0	D.E.C	1	Fx. Pt.	MONTH JAN
.3.9.1	D.E.C	1	Fx. Pt.	DAY 1
.3.9.2	D.E.C	1961	Fx. Pt.	YEAR 1961
.3.9.3	D.E.C	0		TIME FROM MIDNIGHT
OPTIONAL START CONDITIONS				
.3.6.0	D.E.C			VELOCITY AT t_0
.3.6.1	D.E.C			PITCH ANGLE χ_0
SENSE SWITCHES:				SW 1 & SW 3 USED ON EXTENDED SEARCH
.4.1.0	D.E.C		SW 1	0 ⇒ VENUS ≠0 ⇒ MARS ★
.4.1.1	D.E.C		SW 2	0 ⇒ OFFLINE ≠0 ⇒ ONLINE
.4.1.2	D.E.C		SW 3	0 ⇒ INTERPLANETARY ≠0 ⇒ LUNAR ★
. . .	. .		★	SW 1 IS ARBITRARY IF SW 3 IS ≠0
.4.1.3	D.E.C	1.0	SW 4	0 ⇒ NOT POLAR COORD. ≠0 ⇒ POLAR COORD.
.4.1.4	D.E.C	0	SW 5	0 ⇒ NOT CART. COORD. ≠0 ⇒ CART. COORD.
.4.1.5	D.E.C	1.0	SW 6	0 ⇒ FT. & RAD. ≠0 ⇒ M. & DEG.

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MOTOR DATA:				CONFIG.:				
MOTOR 1				MOTOR 2				
LOC	OP			LOC	OP			
.2.3.0	D.E.C	72400.		.2.4.2	D.E.C	7550.		F_o
.2.3.1	D.E.C	5.25		.2.4.3	D.E.C	1.75		F_e
.2.3.2	D.E.C	50000.		.2.4.4	D.E.C	10000.		W_g
.2.3.3	D.E.C	1500.	H1	.2.4.5	D.E.C	640.	H2	W_e
. . .	. .			. . .	. .			
.2.3.4	D.E.C	225.		.2.4.6	D.E.C	130.		$\dot{W}_p$
.2.3.5	D.E.C	50000.		.2.4.7	D.E.C	10000.		W_f
.2.3.6	D.E.C	3.75		.2.4.8	D.E.C	2.		D
.2.3.7	D.E.C	0.3	H1	.2.4.9	D.E.C	0	H2	Cd_i
Δt_b = 171.11 sec				Δt_b = 72 sec				★★
Δt_c = 15 sec				Δt_c = 100 sec				★★
.2.3.8	D.E.C	171.11		.2.5.0	D.E.C	72.		Δt_b
.2.3.9	D.E.C	120		.2.5.1	D.E.C	121		GDB
.2.4.0	D.E.C	15.		.2.5.2	D.E.C	100.		Δt_c
.2.4.1	D.E.C	117	H1	.2.5.3	D.E.C	117	H2	GDC
. . .	. .			. . .	. .			
. . .	. .			. . .	. .			

EXPLANATION OF MOTOR DATA INPUT:

F_o: ±NO. ⇒ VACUMN THRUST CONSTANT Lbs.	FOR PROGRAMMER:
FX.PT. n = -1, -2 ⇒ POLYN. $F_o(t)$ USED	Δt_b: ≥ 0 ⇒ BURNING PERIOD SECS.
F_e: EXHAUST AREA OF THROAT ft.²	< 0 ⇒ BURNING PERIOD DETERMINED
W_g: ≠ 0 ⇒ GROSS WEIGHT Lbs.	BY SHUTOFF EQUATION
= 0 ⇒ W_g COMPUTED FR. PREVIOUS STAGE	= -0 ⇒ FOR UNI-, BIVARIATE SEARCH
W_e: WEIGHT DISCARDED Lbs.	ONLY. ENDS SEARCH PASS AT
$\dot{W}_p$: ±NO. ⇒ WEIGHT FLOW CONSTANT RATE Lbs./sec	SHUTOFF AS Δt_b < 0.
FX.PT. n = -1, -2 ⇒ POLYN. $\dot{W}_p(t)$ USED	GDB: GUIDANCE MODE DURING BURNING.
W_f: FUEL WEIGHT SHUTOFF. USE W_g	FX.PT. A LAMBDA NO. WHICH MAY BE
WHEN OPTION NOT DESIRED Lbs.	CHANGED BY TIME STOPS.
D: EFFECTIVE DIAMETER OF VEHICLE ft.	Δt_c: ≥ 0 ⇒ COASTING PERIOD SECS.
Cd_i: ≠ 0 ⇒ DRAG COEFF. = $Cd_i/\pi/4$	< 0 ⇒ $\lvert\Delta t_c\rvert$ = COAST PERIOD AT END
= 0 ⇒ C_{d_0} COMPUTED BY A SET C(M)	OF COAST COMPUTES TRAJ.
POLYNS. M = MACH NO.	OF EJECTED VEHICLE
★★ STATE BURNING/COAST TIME (SECS)	= -0 ⇒ END OF POWERED PORTION
AND GUIDANCE MODE DESIRED	OF EXTENDED SEARCH
DURING THIS TIME.	GDC: SAME AS GDB BUT FOR COAST

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MOTOR DATA: CONT.			CONFIG. =			
MOTOR 3			MOTOR 4			
LOC	OP	73	LOC	OP	73	
.2.5.4	D.E.C		.2.6.6	D.E.C		F_o
.2.5.5	D.E.C		.2.6.7	D.E.C		F_e
.2.5.6	D.E.C		.2.6.8	D.E.C		ω_q
.2.5.7	D.E.C	M3	.2.6.9	D.E.C	M4	ω_e
. . .	. .		. . .	. .		
.2.5.8	D.E.C		.2.7.0	D.E.C		$\dot{\omega}_p$
.2.5.9	D.E.C		.2.7.1	D.E.C		ω_f
.2.6.0	D.E.C		.2.7.2	D.E.C		D
.2.6.1	D.E.C	M3	.2.7.3	D.E.C	M4	C_{di}
Δt_b =			Δt_b =			**
Δt_c =			Δt_c =			**
.2.6.2	D.E.C		.2.7.4	D.E.C		Δt_b
.2.6.3	D.E.C		.2.7.5	D.E.C		GDB
.2.6.4	D.E.C		.2.7.6	D.E.C		Δt_c
.2.6.5	D.E.C	M3	.2.7.7	D.E.C	M4	GDC
. . .	. .		. . .	. .		
. . .	. .		. . .	. .		
MOTOR 5			MOTOR 6			
LOC	OP	73	LOC	OP	73	
.2.7.8	D.E.C		.2.9.0	D.E.C		F_o
.2.7.9	D.E.C		.2.9.1	D.E.C		F_e
.2.8.0	D.E.C		.2.9.2	D.E.C		ω_q
.2.8.1	D.E.C	M5	.2.9.3	D.E.C	M6	ω_e
. . .	. .		. . .	. .		
.2.8.2	D.E.C		.2.9.4	D.E.C		$\dot{\omega}_p$
.2.8.3	D.E.C		.2.9.5	D.E.C		ω_f
.2.8.4	D.E.C		.2.9.6	D.E.C		D
.2.8.5	D.E.C	M5	.2.9.7	D.E.C	M6	C_{di}
Δt_b =			Δt_b =			**
Δt_c =			Δt_c =			**
.2.8.6	D.E.C		.2.9.8	D.E.C		Δt_b
.2.8.7	D.E.C		.2.9.9	D.E.C		GDB
.2.8.8	D.E.C		.3.0.0	D.E.C		Δt_c
.2.8.9	D.E.C	M5	.3.0.1	D.E.C	M6	GDC
. . .	. .		. . .	. .		
. . .	. .		. . .	. .		

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MOTOR DATA OPTIONS:

THRUST POLYNOMIAL $F_0(t) = f_0 + f_1 t + \cdots + f_6 t^6$

F_0 = −1 code LOC	OP	F_0 = −2 code LOC	OP	
.3.1.0	D.E.C	.3.1.7	D.E.C	f_6
.3.1.1	D.E.C	.3.1.8	D.E.C	f_5
.3.1.2	D.E.C	.3.1.9	D.E.C	f_4
.3.1.3	D.E.C	.3.2.0	D.E.C	f_3
. . .	. .	. . .	. .	
.3.1.4	D.E.C	.3.2.1	D.E.C	f_2
.3.1.5	D.E.C	.3.2.2	D.E.C	f_1
.3.1.6	D.E.C	.3.2.3	D.E.C	f_0

WEIGHT FLOW POLYNOMIAL $\dot{\omega}_p(t) = \omega_0 + \omega_1 t + \cdots + \omega_6 t^6$

$\dot{\omega}_p$ = −1 code LOC	OP	$\dot{\omega}_p$ = −2 code LOC	OP	
.3.3.0	D.E.C	.3.3.7	D.E.C	ω_6
.3.3.1	D.E.C	.3.3.8	D.E.C	ω_5
.3.3.2	D.E.C	.3.3.9	D.E.C	ω_4
.3.3.3	D.E.C	.3.4.0	D.E.C	ω_3
. . .	. .	. . .	. .	
.3.3.4	D.E.C	.3.4.1	D.E.C	ω_2
.3.3.5	D.E.C	.3.4.2	D.E.C	ω_1
.3.3.6	D.E.C	.3.4.3	D.E.C	ω_0

DRAG COEFFICIENT SELECTION $C_{d_2} = 0$

LOC	OP		LOC	OP	
.4.3.5	D.E.C	m_1	.4.4.7	D.E.C	C_3^{III}
.4.3.6	D.E.C	m_2	.4.4.8	D.E.C	C_2
.4.3.7	D.E.C	m_3	.4.4.9	D.E.C	C_1
.4.3.8	D.E.C	m_4	.4.5.0	D.E.C	C_0
. . .	. .		. . .	. .	
.4.3.9	D.E.C	C_3^{I}	.4.5.1	D.E.C	C_3^{IV}
.4.4.0	D.E.C	C_2	.4.5.2	D.E.C	C_2
.4.4.1	D.E.C	C_1	.4.5.3	D.E.C	C_1
.4.4.2	D.E.C	C_0	.4.5.4	D.E.C	C_0
. . .	. .		. . .	. .	
.4.4.3	D.E.C	C_3^{II}	.4.5.5	D.E.C	C_3^{V}
.4.4.4	D.E.C	C_2	.4.5.6	D.E.C	C_2
.4.4.5	D.E.C	C_1	.4.5.7	D.E.C	C_1
.4.4.6	D.E.C	C_0	.4.5.8	D.E.C	C_0

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GUIDANCE PARAMETERS:

CONSTANT CHI

$\bar{\chi}_{i=1,\dots,4}$ **LAM 19**

LOC	OP		
.3.8.2	D.E.C		$\bar{\chi}_1$
.3.8.3	D.E.C		$\bar{\chi}_2$
.3.8.4	D.E.C		$\bar{\chi}_3$
.3.8.5	D.E.C		$\bar{\chi}_4$
		73 X	

PITCH CONTROL FROM HORIZON

$\mu_{i=1,\dots,4}$ **LAM 38**

LOC	OP		
.3.7.0	D.E.C		μ_1
.3.7.1	D.E.C		μ_2
.3.7.2	D.E.C		μ_3
.3.7.3	D.E.C		μ_4
		73 M	

CONSTANT CHANGE IN CHI

$\Delta\chi_{i=1,\dots,3}$ **LAM 21**

LOC	OP		
.3.8.6	D.E.C	1.2	$\Delta\chi_1$
.3.8.7	D.E.C		$\Delta\chi_2$
.3.8.8	D.E.C		$\Delta\chi_3$
. . .	. .		
		73 DX	

CONSTANT CHANGE IN YAW

$\Delta\tau_{i=1,\dots,4}$ **LAM 25**

LOC	OP		
.3.7.8	D.E.C		$\Delta\tau_1$
.3.7.9	D.E.C		$\Delta\tau_2$
.3.8.0	D.E.C		$\Delta\tau_3$
.3.8.1	D.E.C		$\Delta\tau_4$
		73 DT	

ANGLE OF ATTACK IN PITCH

$\alpha_p = p_0$ **LAM 14**

$\alpha_p(t) = p_0 + p_1 t + \cdots + p_6 t^6$ **LAM 15**

LOC	OP		
.4.8.5	D.E.C		p_6
.4.8.6	D.E.C		p_5
.4.8.7	D.E.C		p_4
.4.8.8	D.E.C		p_3
. . .	. .		
.4.8.9	D.E.C		p_2
.4.9.0	D.E.C		p_1
.4.9.1	D.E.C		p_0
		73 AP(T)	

ANGLE OF ATTACK IN YAW

$\alpha_y(t) = y_0 + y_1 t + \cdots + y_6 t^6$ **LAM26**

LOC	OP		
.4.9.2	D.E.C		y_6
.4.9.3	D.E.C		y_5
.4.9.4	D.E.C		y_4
.4.9.5	D.E.C		y_3
. . .	. .		
.4.9.6	D.E.C		y_2
.4.9.7	D.E.C		y_1
.4.9.8	D.E.C		y_0
		73 AY(T)	

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GUIDANCE PARAMETERS: CONT.

PITCH A FUNCTION OF TIME $\quad \psi(t) = C_0 + C_1(t) + \cdots + C_6 t^6 \quad$ LAM 20

MOTOR 1 LOC	OP			MOTOR 2 LOC	OP			
.7.0.0	D.E.C			.7.0.7	D.E.C			C_6
.7.0.1	D.E.C			.7.0.8	D.E.C			C_5
.7.0.2	D.E.C			.7.0.9	D.E.C			C_4
.7.0.3	D.E.C		M1	.7.1.0	D.E.C		M2	C_3
. . .	. .	76 X(T)		. . .	. .	76 X(T)		
.7.0.4	D.E.C	0		.7.1.1	D.E.C			C_2
.7.0.5	D.E.C	−0.1461		.7.1.2	D.E.C			C_1
.7.0.6	D.E.C	25.	M1	.7.1.3	D.E.C		M2	C_0
		76 X(T)				76 X(T)		

$\psi(t)$ CONT

MOTOR 3 LOC	OP			MOTOR 4 LOC	OP			
.7.1.4	D.E.C			.7.2.1	D.E.C			C_6
.7.1.5	D.E.C			.7.2.2	D.E.C			C_5
.7.1.6	D.E.C			.7.2.3	D.E.C			C_4
.7.1.7	D.E.C		M3	.7.2.4	D.E.C		M4	C_3
. . .	. .	76 X(T)		. . .	. .	76 X(T)		
.7.1.8	D.E.C			.7.2.5	D.E.C			C_2
.7.1.9	D.E.C			.7.2.6	D.E.C			C_1
.7.2.0	D.E.C		M3	.7.2.7	D.E.C		M4	C_0
		76 X(T)				76 X(T)		

$\psi(t)$ CONT.

MOTOR 5 LOC	OP			MOTOR 6 LOC	OP			
.7.2.8	D.E.C			.7.3.5	D.E.C			C_6
.7.2.9	D.E.C			.7.3.6	D.E.C			C_5
.7.3.0	D.E.C			.7.3.7	D.E.C			C_4
.7.3.1	D.E.C		M5	.7.3.8	D.E.C		M6	C_3
. . .	. .	76 X(T)		. . .	. .	76 X(T)		
.7.3.2	D.E.C			.7.3.9	D.E.C			C_2
.7.3.3	D.E.C			.7.4.0	D.E.C			C_1
.7.3.4	D.E.C		M5	.7.4.1	D.E.C		M6	C_0
		76 X(T)				76 X(T)		

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GUIDANCE PARAMETERS: CONT.

YAW A FUNCTION OF TIME $\tau(t) = v_0 + v_1(t) + \cdots + v_6 t^6$ LAM 24

MOTOR 1

LOC	OP		73	
.7.5.0	D.E.C			v_6
.7.5.1	D.E.C			v_5
.7.5.2	D.E.C			v_4
.7.5.3	D.E.C		M1	v_3
. . .	. .	76 T(T)		
.7.5.4	D.E.C			v_2
.7.5.5	D.E.C			v_1
.7.5.6	D.E.C		M1	v_0
		76 T(T)		

MOTOR 2

LOC	OP		73	
.7.5.7	D.E.C			v_6
.7.5.8	D.E.C			v_5
.7.5.9	D.E.C			v_4
.7.6.0	D.E.C		M2	v_3
. . .	. .	76 T(T)		
.7.6.1	D.E.C			v_2
.7.6.2	D.E.C			v_1
.7.6.3	D.E.C		M2	v_0
		76 T(T)		

$\tau(t)$ CONT.

MOTOR 3

LOC	OP		73	
.7.6.4	D.E.C			v_6
.7.6.5	D.E.C			v_5
.7.6.6	D.E.C			v_4
.7.6.7	D.E.C		M3	v_3
. . .	. .	76 T(T)		
.7.6.8	D.E.C			v_2
.7.6.9	D.E.C			v_1
.7.7.0	D.E.C		M3	v_0
		76 T(T)		

MOTOR 4

LOC	OP		73	
.7.7.1	D.E.C			v_6
.7.7.2	D.E.C			v_5
.7.7.3	D.E.C			v_4
.7.7.4	D.E.C		M4	v_3
. . .	. .	76 T(T)		
.7.7.5	D.E.C			v_2
.7.7.6	D.E.C			v_1
.7.7.7	D.E.C		M4	v_0
		76 T(T)		

$\tau(t)$ CONT.

MOTOR 5

LOC	OP		73	
.7.7.8	D.E.C			v_6
.7.7.9	D.E.C			v_5
.7.8.0	D.E.C			v_4
.7.8.1	D.E.C		M5	v_3
. . .	. .	76 T(T)		
.7.8.2	D.E.C			v_2
.7.8.3	D.E.C			v_1
.7.8.4	D.E.C		M5	v_0
		76 T(T)		

MOTOR 6

LOC	OP		73	
.7.8.5	D.E.C			v_6
.7.8.6	D.E.C			v_5
.7.8.7	D.E.C			v_4
.7.8.8	D.E.C		M6	v_3
. . .	. .	76 T(T)		
.7.8.9	D.E.C			v_2
.7.9.0	D.E.C			v_1
.7.9.1	D.E.C		M6	v_0
		76 T(T)		

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STATION PRINT INFORMATION:

1. NAME = LOC	OP	2. NAME = LOC	OP	
.6.5.0	D.E.C	.6.5.8	D.E.C	GEOC. LAT.
.6.5.1	D.E.C	.6.5.9	D.E.C	LONG.
.6.5.2	D.E.C	.6.6.0	D.E.C	RADIUS
. . .	. . .	. . .	. .	
.6.5.3	D.E.C	.6.6.1	D.E.C	FREQ
.6.5.4	D.E.C	.6.6.2	D.E.C	A_i
.6.5.5	D.E.C	.6.6.3	D.E.C	B_i
.6.5.6	D.E.C	.6.6.4	D.E.C	C_i
.6.5.7	D.E.C	.6.6.5	D.E.C	D_i

3. NAME = LOC	OP	4. NAME = LOC	OP	
.6.6.6	D.E.C	.6.7.4	D.E.C	GEOC. LAT.
.6.6.7	D.E.C	.6.7.5	D.E.C	LONG.
.6.6.8	D.E.C	.6.7.6	D.E.C	RADIUS
. . .	. . .	. . .	. .	
.6.6.9	D.E.C	.6.7.7	D.E.C	FREQ
.6.7.0	D.E.C	.6.7.8	D.E.C	A_i
.6.7.1	D.E.C	.6.7.9	D.E.C	B_i
.6.7.2	D.E.C	.6.8.0	D.E.C	C_i
.6.7.3	D.E.C	.6.8.1	D.E.C	D_i

5. NAME = LOC	OP	6. NAME = LOC	OP	
.6.8.2	D.E.C	.6.9.0	D.E.C	GEOC. LAT.
.6.8.3	D.E.C	.6.9.1	D.E.C	LONG.
.6.8.4	D.E.C	.6.9.2	D.E.C	RADIUS
. . .	. . .	. . .	. .	
.6.8.5	D.E.C	.6.9.3	D.E.C	FREQ
.6.8.6	D.E.C	.6.9.4	D.E.C	A_i
.6.8.7	D.E.C	.6.9.5	D.E.C	B_i
.6.8.8	D.E.C	.6.9.6	D.E.C	C_i
.6.8.9	D.E.C	.6.9.7	D.E.C	D_i

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SHUTOFF EQUATIONS

VELOCITY FORM $V_x = K_1 \int_{t_0}^{t} a_x \, dt + V_x(t_0) + K_2 + K_3(t-a) + K_4(t-b)^2$ $V_\Delta = V_{xs} - V_x$

LOC	OP		LOC	OP		
5.0	DEC		5.8	DEC		K_1
5.1	DEC		5.9	DEC		K_2
5.2	DEC		6.0	DEC		K_3
5.3	DEC		6.1	DEC		K_4
...	..		...	..		
5.4	DEC		6.2	DEC		a
5.5	DEC		6.3	DEC		b
5.6	DEC		6.4	DEC		$V_x(t_0)$
5.7	DEC		6.5	DEC		V_{xs}

GNG-06-9

SEARCH:		IND. VARS. =					
						EXPLANATION	
LOC 6	7	8 . . . 10	11	12	30	UNI-, BIVARIATE	EXTENDED
. . 7 0		D.E.C					* ITERATIONS = i
. . 7 1		D.E.C					* DIMENSION = n
INDEPENDENT VARIABLES							
. . 7 2		D.E.C			LØC	* LOC IND VAR$_1$	* LOC IND VAR$_1$
. . 7 3		D.E.C				* LOC IND VAR$_2$	* LOC IND VAR$_2$ IF IXI ON T_L = LOC T_L
. . 7 4		D.E.C					* LOC IND VAR$_3$
. . 7 5		D.E.C					* LOC IND VAR$_4$
DECREMENT FOR IND. VARIABLES							
. . 8 2		D.E.C			D	Δ IND. VAR$_1$	Δ IND VAR$_1$
. . 8 3		D.E.C				Δ IND. VAR$_2$	Δ IND. VAR$_2$
. . 8 4		D.E.C					Δ IND. VAR$_3$
. . 8 5		D.E.C					Δ IND. VAR$_4$
MAXIMUM DECREMENT ALLOWED IND. VARS.							
. . 9 0		D.E.C			MAXD		MAXD. IND. VAR$_1$
. . 9 1		D.E.C					MAXD. IND. VAR$_2$
. . 9 2		D.E.C					MAXD. IND. VAR$_3$
. . 9 3		D.E.C					MAXD. IND. VAR$_4$
DEPENDENT VARIABLES							
. . 7 6		D.E.C			LØC	* LOC DEP. VAR$_1$	
. . 7 7		D.E.C				* LOC. DEP. VAR$_2$	
DESIRED VALUES FOR DEP. VARIABLES							
. . 7 8		D.E.C			VAL	DEP. VAR$_1$	A
. . 7 9		D.E.C				DEP. VAR$_2$	B
. . 8 0		D.E.C					C
. . 8 1		D.E.C					J.D. IMPACT
ERRORS ALLOWED IN DEP. VARIABLES							
. . 8 6		D.E.C			ER	DEP. VAR$_1$	A^*
. . 8 7		D.E.C				DEP. VAR$_2$	B^*
. . 8 8		D.E.C					C^*
. . 8 9		D.E.C					TIME BOUND
FOR IXI ON T_L / PARTIALS CODE							
. . 9 4		D.E.C					Δt_L
. . 9 5		D.E.C					MAXD. t_L
. . 9 6		D.E.C					* ITERATIONS = I
. . 9 7		D.E.C					* TYPE {0 = RECOMP., 1 = SAVE; WITH IXI {2 = RECOMP., 3 = SAVE
. . .		. .					

* ⟹ FIXED POINT NUMBERS

GNG-06-10

TIME STOPS:

LOC	OP	TIME, LAMXX	COMMENTS
150	DEC	150., 110	NO PRINT OUT
152	DEC	398., 107	2 SEC. PRINT
154	DEC	398., 134	PRINT CONICS
156	DEC	410., 112	P.C. AND HALT
158	DEC		
160	DEC		
162	DEC		
164	DEC		

GNG-06-11

POLAR COORDINATES:			SW4 ≠ 0	EXPLANATION
LOC	OP			SINGLE VALUED TABLE
.5.0.0	D.E.C	55.		σ_{LS} = LAUNCH SIGMA
.5.0.1	D.E.C	0		σ_f = 0
.5.0.2	D.E.C	0	T.	$\Delta\sigma$ = 0 ⟹ NOT MULTIVALUED TABLE
. . .	. .			
.5.0.3	D.E.C	6421132.	R	R = RADIUS
.5.2.3	D.E.C	30.45	PHI	Φ = GEOCENTRIC LATITUDE
.5.4.3	D.E.C	2400.	V	V = VELOCITY
.5.6.3	D.E.C	27.98	GAMMA	γ = PITCH ANGLE
.5.8.3	D.E.C	56.78	SIGMA	σ = AZIMUTH ANGLE
.6.0.3	D.E.C	280.02	THETA	θ = LONGITUDE
. . .	T.R.A	3,4		

CARTESIAN COORDINATES:				SW5≠0	EXPLANATION
LOC	OP	CASE 1	CASE 2	CASE 3	
.3.5.0	D.E.C				ID NO. XXX.XXX
.3.6.3	**D.E.C**				X_P
.3.6.4	D.E.C				Y_P
.3.6.5	D.E.C				Z_P
. . .	. .				
.3.6.6	**D.E.C**				$\dot{X}_P$
.3.6.7	D.E.C				$\dot{Y}_P$
.3.6.8	D.E.C				$\dot{Z}_P$
. . .	. .				
. . .	T.R.A	3,4	3,4	3,4	
. . .	. .	CASE 4	CASE 5	CASE 6	
.3.5.0	D.E.C				ID.NO. XXX.XXX
.3.6.3	**D.E.C**				X_P
.3.6.4	D.E.C				Y_P
.3.6.5	D.E.C				Z_P
. . .	. .				
.3.6.6	**D.E.C**				$\dot{X}_P$
.3.6.7	D.E.C				$\dot{Y}_P$
.3.6.8	D.E.C				$\dot{Z}_P$
. . .	. .				
. . .	T.R.A	3,4	3,4	3,4	

CNG-06-12

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POWER FLIGHT TRAJECTORY                                                          PAGE  1
IDENT   1.000            RUN                 DATE  5.00                          GNG06

LAUNCH CONDITIONS
AZI  2 54999998 LAT  2 27540086 GED  2 27999999 LON  3 27899999 RAD  7 63736855
LAUNCH DATE   1  1 1961 JDT  7 24372005 GHA  3 10040257 TL     00000000

CHI POLY COEFF.        2 250000    -140100     000000    000000    000000    000000    000000

MOTOR  FRC 5 72400000 FE  1 52500000 WG  5 50000000 APD 3 215000 DIA 1 375000 CD   30000000 BRN 3 171110 CST 2 150000

TIM 3 15000000 ACC 2 14120097 INA   00000000 DDX 2 12364199 DDY 1-36877448 DDZ   21388422 DT  1 10000000 DTP   00000000
XP  6 29856064 YP  5 39583305 ZP  6-14905784 DXP 4 25012450 DYP 4 10199563 DZP 3 24046970 V   4 27115922 GAM 2 14532238
XM    00000000 YM    00000000 ZM    00000000 DXM   00000000 DYM   00000000 DZM   00000000 VE  4 24000000 FTH 2 27979999
ARC 1 29797953 ALT 5 48280492 R   7 64211319 XDD 2 12806217 YDD 1 59716368 ZDD   00000000 LAT 2 30449999 LON 3 28001991
M   5 49999999 F   5 72388066 A   7 34129834 N     00000000 CHI 2 24999999 TAU   00000000 SIG 2 56780003 SGI 2 61922760
XA  7 51659656 VA  7 19885422 ZA  7 32541413 DXA 3-42525387 DYA 4 21687474 DZA 4 15716417 ALA   52957223 FAV 5 48500000

TIM 3 15000000 ACC 2 14130097 INA   00000000 DDX 2 12364199 DDY 1-36877448 DDZ   21388422 DT  1 20000000 DTP 1 20000000
XP  6 29856064 YP  5 39583305 ZP  6-14905784 DXP 4 25012450 DYP 4 10199563 DZP 3 24046970 V   4 27118926 GAM 2 14532238
XM    00000000 YM    00000000 ZM    00000000 DXM   00000000 DYM   00000000 DZM   00000000 VE  4 24000000 FTH 2 27979999
ARC 1 29797953 ALT 5 48280492 R   7 64211719 XDD 2 12806217 YDD 1 59716368 ZDD   00000000 LAT 2 30449991 LON 3 28001991
M   5 49999999 F   5 72388066 A   7 34129834 N     00000000 CHI 2 24999999 TAU   00000000 SIG 2 56780003 SGI 2 61922760
XA  7 51659656 VA  7 19885422 ZA  7 32541413 DXA 3-42525387 DYA 4 21687474 DZA 4 15716417 ALA   52957223 FAV 5 48500000

TIM 3 32110999 ACC-10-95068700 INA 4 40707972 DDX 1-17877212 DDY 1-90386412 DDZ   13651807 DT  1 20000000 DTP 1 20000000
XP  7 10073420 YP  6 15231507 ZP  6-10510438 DXP 4 68945416 DYP 3 17998068 DZP 3 27037361 V   4 59021879 GAM 2 10142285
XM  6 28000095 YM  5 70040676 ZM    00000000 DXM 4 45378450 DYM 3 76203562 DZM   00000000 VE  4 65577577 FTH 2 10681080
ARC 1 88263329 ALT 6 23067714 R   7 66079149 XD-10-95109416 YD-11-24828170 ZDD   00000000 LAT 2 33397904 LON 3 25563412
M   5 11500251 F     00000000 A  -8 11100759 N     00000000 CHI 1 14553529 TAU   00000000 SIG 2 59774454 SGI 2 61447128
XA  7 48949385 VA  7 25352245 ZA  7 36743607 DXA 4-24311004 DYA 4 49439487 DZA 4 33806064 ALA 1 10778365 FAV 5 10000251

COAST CONDITIONS

PITCH UP   1 12000000

MOTOR  FRC 4 75500000 FE  1 17500000 WG  5 10000000 WPD 3 130000 DIA 1 200000 CD POLYNOMIAL  FN 2 720000 CST 3 100000

TIM 3 33610999 ACC 1 74031700 INA   00000000 DDX 1 58827174 DDY 1-87636710 DDZ   12990588 DT  2 10000000 DTP 2 10000000
XP  7 11105991 YP  6 15420092 ZP  6-10199217 DXP 4 68707445 DYP 3 44927466 DZP 3 27037165 V   4 67782992 GAM 1 99041787
XM  6 35896862 YM  5 83371169 ZM    00000000 DXM 4 45378451 DYM 3 76203561 DZM   00000000 VE  4 65725431 FTH 2 10434047
ARC 1 90998052 ALT 6 24861079 R   7 66205138 XDD 1 74098833 YDD   20344463 ZDD   00000000 LAT 2 33516025 LON 3 23650328
M   4 99999499 F   4 75500000 A     00000000 N     00000000 CHI 1 15746222 TAU   00000000 SIG 2 60278880 SGI 2 61960667
XA  7 48424134 YA  7 26068803 ZA  7 36345071 DXA 4-75318594 DYA 4 48003760 DZA 4 33047174 ALA   82488899 FAV 4 92600000

TIM 3 40000000 ACC 2 43677595 INA 4 10110070 DDX 2 41806688 DDY 1-74792749 DDZ   10241186 DT  1 20000000 DTP 1 20000000
XP  7 15694253 YP  6 13982111 ZP  5-82444551 DXP 4 77687948 DYP 3-49000019 DZP 3 27978587 V   4 77892525 GAM 1 98243489
XM  6 67209173 YM  6 13769539 ZM    00000000 DXM 4 55485708 DYM 3 78981861 DZM   00000000 VE  4 74395260 FTH 2 10291622
ARC 2 13573384 ALT 6 32657617 R   7 68976691 XDD 2 43681098 YDD 1 12007612 ZDD   00000000 LAT 2 35583151 LON 3 29040290
M   4 16942988 F   4 75500000 A     00000000 N     00000000 CHI 1 15740222 TAU   00000000 SIG 2 62712265 SGI 2 64073445
XA  7 45918944 YA  7 29269474 ZA  7 38872848 DXA 4-44475579 DYA 4 53505007 DZA 4 35019755 ALA 1 47902277 FAV 4 10542588

   CONIC - ELLIPSE
   MOM 11 51604735 ENR  8-29179327 ECC   17170846 HCS  7 68304412 PER -1 05022277 ANM  2 93420465
   APO  7 80032857 FOE  7 56575067 VAP  4 54229487 VFG  4 90859611 TPG  4 11505029 EAN  2 83409852
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PAGE 2

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TIM 3 40200000 ACX 2 51618803 INA 4 11059451 DDX 2 49506712 DDY 1-72098724 DDZ   10155833 DT   1 20000000 DTF 1 20000000
XP  7 15850510 VP  6 13812029 ZP  5-82985768 DXP 4 78594595 DYP 3-50472143 DZP 3 27099284 V    4 78806245 GAM 1 08923728
XM  6 68328100 YM  6 13927756 ZM    00000000 DXM 4 56433725 DYM 3 79242574 DZM   00000000 VE   4 75308658 PTH 2 10358781
ARC 2 13704872 ALT 6 32927891 R   7 67003504 XDD 2 51570312 YDD 1 14184270 ZDD   00000000 LAT  2 35640790 LON 3 [illegible]
M   4 14342988 F   4 75500000 A     00000000 N     00000000 CHI 1 15746222 TAU   00000000 SIG  2 62900250 SGI 2 64120132
XA  7 45829407 VA  7 29407044 ZA  7 39043064 DXA 4-45074416 DYA 4 54082792 DZA 4 35408660 ALA  1 48559777 PAV 3 [illegible]

    CONIC - ELLIPSE
    MOM 11 52017882 ENR 8-28439018 ECC   17716110 AXS 7 70081621 PER -1 67575561 ANM 2 85759482
    APO  7 82496651 PGE 7 57665380 VAP 4 83054570 VPG 4 90206413 TPG  4 10673875 EAN 2 75048255

TIM 3 40400000 ACX 2 63047659 INA 4 12198522 DDX 2 30912894 DDY 1-68878558 DDZ   10070379 DT   1 20000000 DTF 1 20000000
XP  7 16008757 VP  6 13760259 ZP  5-82325580 DXP 4 70691200 DYP 3-51583007 DZP 3 28210511 V    4 79909056 GAM 1 99758463
XM  6 69467780 YM  6 14036543 ZM    00000000 DXM 4 57571766 DYM 3 79555579 DZM   00000000 VE   4 70411380 PTH 2 10437400
ARC 2 13837973 ALT 6 33207860 R   7 67030873 XDD 2 [illegible] YDD 1 17324792 ZDD 00000000 LAT  2 35699022 LON 3 21074031
M   4 11742988 F   4 75500000 A     00000000 N     00000000 CHI 1 15746222 TAU   00000000 SIG  2 62880671 SGI 2 64130112
XA  7 45738584 VA  7 29515992 ZA  7 29114349 DXA 4-45766000 DYA 4 54191971 DZA 4 35896295 ALA  1 49077250 PAV 3 [illegible]

    CONIC - ELLIPSE
    MOM 11 52753901 ENR 8-27540167 ECC   13785370 AXS 7 [illegible] PER -1 70912952 ANM 2 77221500
    APO  7 85964767 PGE 7 59774976 VAP 4 81366864 VPG 4 80755774 TPG  3 [illegible] EAN 2 [illegible]

TIM 3 40600000 ACX 2 80976576 INA 4 136[illegible] DDX 2 78816476 DDY 1-63789471 DDZ-1 99647712 DT 1 20000000 DTF 1 20000000
XP  7 16169462 VP  6 13655144 ZP  5-81764[illegible] DXP 4 81073000 DYP 3-57213981 DZP 3 [illegible] V 4 [illegible] GAM 1 [illegible]
XM  6 70632879 YM  6 14246030 ZM    00000000 DXM 4 58997190 DYM 3 [illegible] DZM 00000000 VE 4 [illegible] PTH 2 10540015
ARC 2 13973067 ALT 6 33486332 R   7 [illegible] XDD 2 [illegible] YDD 1 2225145[illegible] ZDD 00000000 LAT 2 [illegible] LON 3 [illegible]
M   3 91429582 F   4 75500000 A     00000000 N     00000000 CHI 1 15746222 TAU   00000000 SIG  2 [illegible] SGI 2 [illegible]
XA  7 45646239 VA  7 29626342 ZA  7 [illegible] DXA 4-[illegible] DYA 4 [illegible] DZA 4 [illegible] ALA 1 [illegible] PAV 3 [illegible]

    CONIC - ELLIPSE
    MOM 11 53674737 ENR 8-26397011 ECC   20600955 AXS 7 [illegible] PER -1 [illegible] ANM 2 [illegible]
    APO  7 91118836 PGE 7 59[illegible] VAP 4 58906298 VPG 4 [illegible] TPG 3 85773471 EAN 2 57260042

TIM 3 40800000 ACX 3 11215440 INA 4 155[illegible] DDX 2 [illegible] DDY 1-[illegible] DDZ-1 [illegible] DT 1 20000000 DTF 1 20000000
XP  7 16333365 VP  6 13547491 ZP  5-[illegible] DXP 4 [illegible] DYP 3-[illegible] DZP 3 [illegible] V 4 83181510 GAM 1 1022[illegible]
XM  6 71830900 YM  6 14476418 ZM    00000000 DXM 4 [illegible] DYM 3 [illegible] DZM 00000000 VE 4 [illegible] PTH 2 [illegible]
ARC 2 14110754 ALT 6 33778049 R   7 [illegible] XDD 2 [illegible] YDD 1 [illegible] ZDD 00000000 LAT 2 [illegible] LON 3 [illegible]
M   3 65429282 F   4 75500000 A     00000000 N     00000000 CHI 1 15746222 TAU   00000000 SIG  2 [illegible] SGI 2 [illegible]
XA  7 45552002 VA  7 29738903 ZA  7 [illegible] DXA 4-[illegible] DYA 4 [illegible] DZA 4 37419072 ALA 1 [illegible] PAV 3 [illegible]

    CONIC - ELLIPSE
    MOM 11 54905650 ENR 8-24837725 ECC [illegible] AXS 7 [illegible] PER -1 [illegible] ANM 2 [illegible]
    APO  7 99490641 PGE 7 60997284 VAP 4 5512[illegible] VPG 4 [illegible] TPG 3 73219711 EAN 2 [illegible]

TIM 3 40010999 ACX   00000000 INA 4 1565[illegible] DDX 1-[illegible] DDY 1-[illegible] DDZ-1 [illegible] DT 1 [illegible] DTF 1 [illegible]
XP  7 16342494 VP  6 13541503 ZP  5-81177135 DXP 4 [illegible] DYP 3-5446[illegible] DZP 3 28060[illegible] V 4 [illegible] GAM 1 [illegible]
XM  6 71897858 YM  6 14415272 ZM    00000000 DXM 4 [illegible] DYM 3 [illegible] DZM 00000000 VE 4 [illegible] PTH 2 [illegible]
ARC 2 14118421 ALT 6 33794424 R   7 [illegible] XDD   00000000 YDD -[illegible] ZDD 00000000 LAT 2 [illegible] LON 3 [illegible]
M   3 63999975 F     00000000 A     00000000 N     00000000 CHI [illegible] TAU 00000000 SIG 2 [illegible] SGI 2 [illegible]
XA  7 45546752 VA  7 29745170 ZA  7 [illegible] DXA 4-4775[illegible] DYA 4 [illegible] DZA 4 [illegible] ALA 1 [illegible] PAV-3 [illegible]

    CONIC - ELLIPSE
    MOM 11 549[illegible] ENR 8-24733512 ECC [illegible] AXS 7 [illegible] PER -1 [illegible] ANM 2 [illegible]
    APO  8 10010603 PGE 7 61[illegible] VAP 4 [illegible] VPG 4 [illegible] TPG 3 72[illegible] EAN 2 [illegible]

COAST CONDITIONS
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                                                                                                   PAGE  3

TIM 3 41000000 ACX    00000000 INA 4 15655472 DDX 1-21374000 DDY 1-85786161 DDZ-1 98129592 DT  2 10000000 DTF 2 10000000
XP  7 16499438 YP   6 13437021 ZP   5-80642613 DXP 4 37016512 DYP 3-56092297 DZP 3 28079161 V    4 83255156 GAM 2 10250587
XM  6 73051290 YM   6 14567428 ZM     00000000 DXM 4 61023013 DYM 3 80505513 DZM    00000000 VE   4 79757246 PTH 2 10705336
APC 2 14250189 ALT  6 34076281 R    7 67117496 XDD    00000000 YDD  -00000000 ZDD    00000000 LAT 2 35878564 LON 3 29117477
M   3 63999075 F      00000000 A      00000000 N      00000000 CHI 3 35613463 TAU    00000000 SIG 2 63133200 SGI 2 64386652
XA  7 45456387 YA   7 29852867 ZA   7 39375502 DXA 4-47368755 DYA 4 56945402 DZA 4 37379472 HLA 1 49387985 PAN-3-24414063

   CONIC - ELLIPSE
   MOM 11 54986887 ENR  8-24733594 ECC    24228499 AXS  7 60581798 PER -1 83319081 ANM  2 57513066
   APO  8 10010556 PGE  7 61058038 VAP  4 54928903 VPG  4 90056752 TPG  3 72679534 EAN  2 46398646

TRAJECTORY COMPLETE
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